Topic 1:

Course Background

(or: Why You're Here, and What You Learned to Get Here)

What Is Discrete Math?

Definition: Discrete Mathematics

. . . .

Contrast this with 'the calculus,' which was developed by Newton and Leibniz to study objects in motion. As a result:

- 'The Calculus' tends to focus on real values
- Discrete Mathematics tends to focus on integer values

Sample Discrete Math Topics

Topics that fall under the umbrella of discrete math include:

- Integral Functions and Relations
- Sets
- Sequences and Summations
- Counting (Permutations/Combinations, etc.)
- Discrete Probability

To understand those, you also need:

- First-Order Logic
- Logical Arguments
- Proof Techniques
- . . . and a fair amount of pre-calculus mathematics

"But Why Do I Have To Take Discrete Math?"

Discrete Structures is an ACM/IEEE core curriculum topic

- See:

  https://www.acm.org/binaries/content/assets/education/cs2013_web_final.pdf

DM topics underlie much of Computer Science, including:

- Logic $\longrightarrow$ Knowledge Representation, Reasoning,
  Natural Language Processing, Computer Architecture
- Proof Techniques $\longrightarrow$ Algorithm Design, Code Verification
- Relations $\longrightarrow$ Database Systems
- Functions $\longrightarrow$ Hashing, Programming Languages
- Recurrence Relations $\longrightarrow$ Recursive Algorithm Analysis
- Probability $\longrightarrow$ Algorithm Design, Simulation

Topics You May Need To Review

- Mathematical concepts, including, but not limited to:
  - Fractions
  - Rational Numbers
  - Basics of Sets
  - Associative, Commutative, Distributive, and Transitive Laws
  - Properties of Inequalities
  - Summation and Product Notation
  - Integer Division (Modulo, Divides, and Congruences)
  - Even and Odd Integers
  - Logarithms and Exponents
  - Positional Number Systems

The Math Review appendix (available from the class web page) can help you review these topics.

Notations for Sets of Values

$\mathbb{Z}$	All integers	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{Z}^+, \mathbb{N}^+$	All positive integers	$\{1, 2, 3, \ldots\}$
$\mathbb{Z}^*, \mathbb{N}_0$	The non–negative integers	$\{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}^{\text{even}}$	Even integers	$\{\ldots, -4, -2, 0, 2, 4, \ldots\}$
$\mathbb{Z}^{\text{odd}}$	Odd integers	$\{\ldots, -3, -1, 1, 3, \ldots\}$
$\mathbb{Q}$	Rational numbers	${}^a/_b,\ a, b \in \mathbb{Z}, b \neq 0$
$\overline{\mathbb{Q}}$	Irrational Numbers	$\{i \mid i \notin Q\}$
$\mathbb{R}$	The real values	$\{\mathbb{Q} \cup \overline{\mathbb{Q}}\}$

Note: Avoid the term “natural numbers” and the plain $\mathbb{N}$.

Commutativity

Assume that $\triangle$ is a binary operator on a set of values S.

If $x \triangle y = y \triangle x$ for any elements x and y in S,

then $\triangle$ is a *commutative* operator.

Example(s):

Addition is commutative on $\mathbb{R}$:

Subtraction is non–commutative on $\mathbb{R}$:

Associativity

Assume that $\triangle$ is a binary operator on a set of values S.

If $(x \triangle y) \triangle z = x \triangle (y \triangle z)$ for any x, y, z in S,

then $\triangle$ is an *associative* operator.

Example(s):

Multiplication is associative on $\mathbb{Z}$:

Subtraction is not associative on $\mathbb{Z}$:

Distributivity (1 / 2)

Assume that $\triangle$ and $\Box$ are binary operators on a set S, and that a, b, c are all values of S.

$\triangle$ is *left–distributive* over $\Box$ when $a \triangle (b \Box c) = (a \triangle b) \Box (a \triangle c)$

$\triangle$ is *right–distributive* over $\Box$ when $(b \Box c) \triangle a = (b \triangle a) \Box (c \triangle a)$

Distributivity (2 / 2)

Example(s):

Multiplication distributes over addition:

This knowledge can help you do large products by hand:

Transitivity

Assume that $\diamond$ defines a relationship on values from S.

For any x, y, z in S, $\diamond$ is *transitive* if whenever $x \diamond y$ and $y \diamond z$, then $x \diamond z$.

Example(s):

"Greater than" is transitive on $\mathbb{R}$:

In sports, "defeats" is not transitive on a set of teams:

Three Fraction Reminders

① The product of fractions is the ratio of the products of the numerators over the products of the denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

② One fraction divided by another equals the product of the numerator fraction and the reciprocal of the denominator:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

③ Computing the sum of two fractions requires a common denominator, then we add the numerators:

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{b}{b} \cdot \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

Rational and Irrational Numbers

Definition: Rational Number

. . . .

Example(s):

A real number that is not rational is **irrational**.

Example(s):

Basic Set Operators (1 / 2)

1. **Union** $(\cup)$**:** $A \cup B$ contains all elements of both set A and set B
2. **Intersection** $(\cap)$**:** $C \cap D$ contains only the elements present in both sets C and D
3. **Difference** $(-)$**:** $E - F$ contains only the elements of set E that are **not** also in set F

   (**Note:** Take out the "not," and you've got a definition for $\cap$)
4. **Complement** $(\overline{\square})$**:** Given a set G, $\overline{G} = \mathcal{U} - G$, the set of available items. where $\mathcal{U}$ is the *universe*.

Note: $X - Y = X \cap \overline{Y}$

Basic Set Operators (2 / 2)

Example(s):

$A = \{1, 2, 4, 9\}$

$B = \{0, 2, 6, 8\}$

$C = \{2, 4, 7\}$

Summation and Product Notation

$$\sum_{i=1}^{5} 2i = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 30$$

where:

- Σ is the ______________________________.
- i is the ______________________________.
- 1 is the ______________________________.
- 5 is the ______________________________.
- $2i$ is the ______________________________.

Summation and Product Notation (cont.)

Switch Σ to Π (capital Pi) for multiplication:

Example(s):

Use parentheses to eliminate confusion:

Example(s):

Nested Summations and Products

Much like nested `FOR` loops.

Example(s):

Modulo and Divides

Integer Division $(\backslash)$ produces quotients;

Modulo $(\%)$ produces remainders

Example(s):

Modulo and Divides (cont.)

Definition: Divides

. . . .

Example(s):

Congruences

Definition: Congruent Modulo m

. . . .

. . . .

(b is called the *base*, r is the *residue* or *remainder*, and m is the *modulus*)

Example(s):

Laws of Exponents

1. $w^{x+y} = w^x w^y$
2. $(w^x)^y = w^{xy}$
3. $v^x w^x = (vw)^x$
4. $\frac{w^x}{w^y} = w^{x-y}$
5. $\frac{v^x}{w^x} = \left(\frac{v}{w}\right)^x$

Laws of Logarithms

The connection between exponents and logarithms:

If $b^y = x$, then $log_b x = y$.

For each of the following laws, $a, b > 0$ and $a, b \neq 1$:

1. $log_a x = \frac{log_b x}{log_b a}$
2. If $m > n > 0$, then $log_b m > log_b n$
3. $b^{log_b x} = x$
4. $log_b(x^y) = y log_b x$
5. $log_b(xy) = log_b x + log_b y$
6. $log_b(\frac{x}{y}) = log_b x - log_b y$

Number Systems: Decimal

The Base 10 (a.k.a. Decimal, a.k.a. Arabic) System

- 10 symbols (glyphs): 0,1,2,3,4,5,6,7,8 and 9.
- In a string of symbols, each position is worth the product of the symbol's value and a power of 10, starting with $10^0 = 1$ on the right.

Example(s):

Number Systems: Binary

The Base 2 (a.k.a. Binary) System

- Just 2 symbols: 0 and 1.
- Each position is valued with increasing powers of 2.

Example(s):

Converting Decimal to Binary

1. Repeated divisions by 2

   Example(s):

2. Sums of powers of 2

   Example(s):

Number Systems: Octal

(Key point: Octal is based on groups of 3 binary digits)

The Base 8 (a.k.a. Octal) System

- 8 symbols: 0 through 7, inclusive
- Each position is valued with increasing powers of 8

Example(s):

Converting Octal to . . .

. . . Decimal: Multiply digits by powers of 8:

Example(s):

. . . Binary: Convert digits to binary, and “degroup:”

Example(s):

Number Systems: Hexadecimal

(Key point: ‘Hex’ is based on groups of 4 binary digits)

The Base 16 (a.k.a. Hexadecimal) System

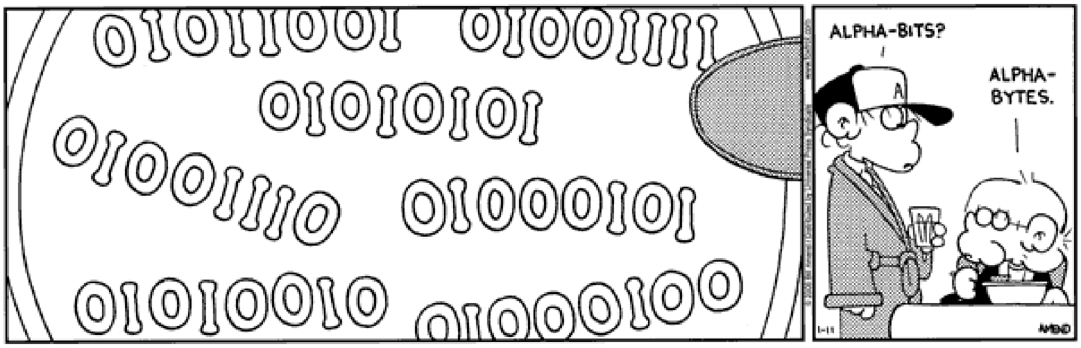
- 16 symbols: 0-9, inclusive, and A-F, inclusive
- Each position is valued with increasing powers of 16

Example(s):

Why Hexadecimal Is More Common Than Octal

What's The Secret Message?

"Foxtrot" from January 11, 2006:



Hint: Find an ASCII table!

Remember!

The math review topics are used in this class, and direct questions about them will be asked on quizzes, Exam #1, and the Final Exam.

If you are not confident in your knowledge of them:

- Read Appendix A in "Kneel Before $\mathbb{Z}^{odd}$,"
- Attend a Supplemental Instruction (SI) session, **and**
- Review and self–test the topics on your own!