

# Topic 5:

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Direct Proofs of  $p \rightarrow q$

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## Handful O' Definitions (1 / 2)

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**Definition: Conjecture**

**Definition: Theorem**

**Definition: Proof**

# Handful O' Definitions (2 / 2)

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## Definition: Lemma

## Definition: Corollary

## Example(s):

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# Why do People Fear Proofs?

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1. Proofs don't come from an assembly line.
  - ▶ Need knowledge, persistence, and creativity
  
2. Creating proofs seems like magic.
  - ▶ But they are systematic in many ways
  
3. Proofs are hard to read and understand.
  - ▶ Only if the writer makes them so
  
4. Institutionalized Fear.
  - ▶ Many teachers avoid them in classes

# Constructing a proof? Remember:

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1. There are several proof techniques for a reason.
  - ▶ One may be easier to use than the others
2. Knowledge of mathematics is important.
  - ▶ Remember our Math Review?
3. There are “tricks” to know.
  - ▶ Ex: Dividing an even # in half leaves no remainder
4. Practice helps . . . a lot!
  - ▶ Just as it does for most everything else
5. Dead ends are expected.
  - ▶ Proofs in books are the final, polished versions

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## Types Of Proof In This Class

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1. Direct Proof
  - ▶ The most common variety
2. Proof by Contraposition
  - ▶ Like Direct, but with a twist
3. Proof by Contradiction
  - ▶ A dark road on a foggy night

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## Our First Conjecture

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**Conjecture:** If  $n$  is even, then  $n^2$  is also even,  $n \in \mathbb{Z}$ .



# Proof-Writing Miscellanea

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- Remember: A conjecture isn't a theorem until proven.
- Don't lose sight of your destination.
- When writing proofs in this class:
  1. Always start with "Proof (*style*):"
  2. Stating your allowed assumptions can help.
  3. Define all introduced variables.
  4. End proofs with "Therefore, " and the conjecture.

[Outside of this class: "Q.E.D." (*quod erat demonstrandum*, Latin for "this was to be demonstrated.")]

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## A Conjecture About Inequalities

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**Conjecture:** If  $0 < a < b$ , then  $a^2 < b^2$ ,  $a, b \in \mathbb{R}$ .

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# “Proof By Cases”

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**Question:** How would you prove that  $\forall x C(x)$  is true, where  $x \in \{6, 28, 496\}$ ?

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## A Direct Proof Employing Cases

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**Conjecture:**  $s \rightarrow r \equiv \neg r \rightarrow \neg s$

Proof (direct): Consider all possible combinations of values of  $r$  and  $s$ :

	$r$	$s$	$s \rightarrow r$	$\neg r \rightarrow \neg s$
Case 1:	T	T	<b>T</b>	<b>T</b>
Case 2:	T	F	<b>T</b>	<b>T</b>
Case 3:	F	T	<b>F</b>	<b>F</b>
Case 4:	F	F	<b>T</b>	<b>T</b>

Therefore,  $s \rightarrow r \equiv \neg r \rightarrow \neg s$ .

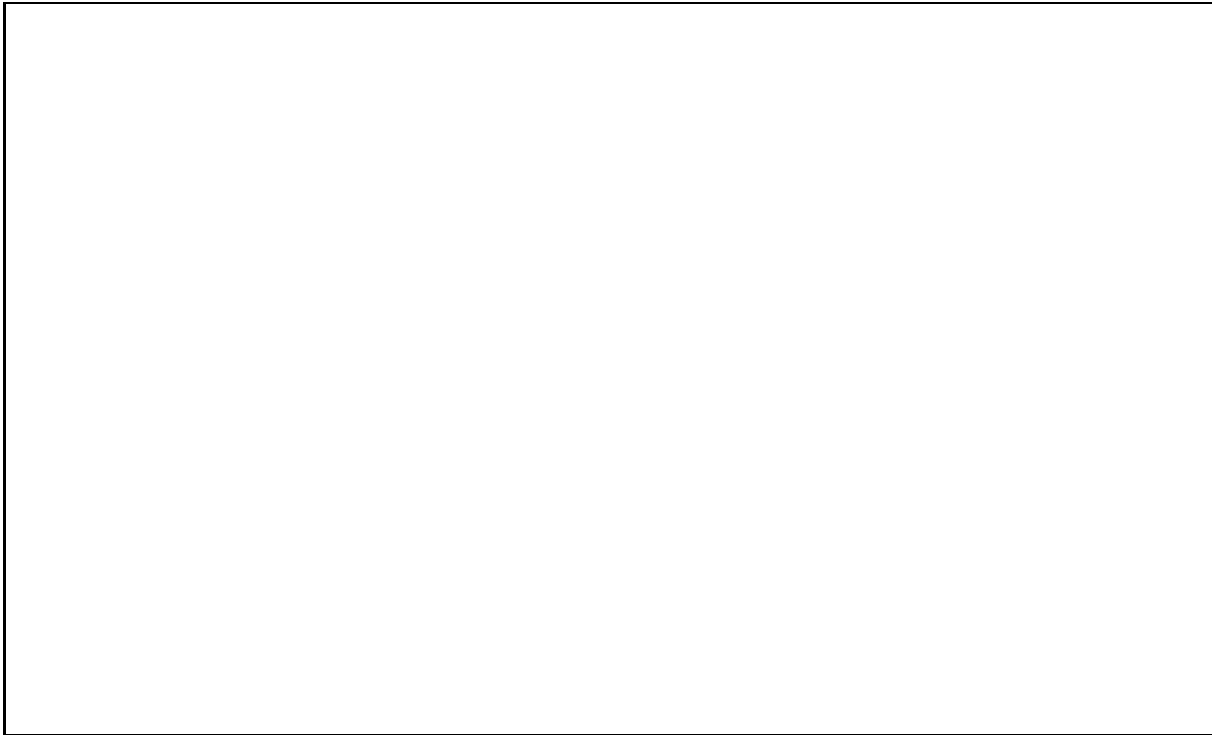
(Yes, this truth table is a direct proof by cases.)

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# A More Interesting Direct Proof With Cases

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**Conjecture:**  $x^2 \% 4 \in \{0, 1\}, x \in \mathbb{Z}$ .



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## Poor Arguments $\longrightarrow$ Poor Proofs (1 / 2)

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**Conjecture:**  $1 < 0$ .

Proof or Goof?:

Consider  $x$  such that  $0 < x < 1$ . Take the base-10 logarithm of both sides of  $x < 1$ :  $\log_{10}x < \log_{10}1$ . By definition,  $\log_{10}1 = 0$ . Divide both sides by  $\log_{10}x$ :

$$\frac{\log_{10}x}{\log_{10}x} < \frac{0}{\log_{10}x}, \text{ which reduces to } 1 < 0.$$

Therefore,  $1 < 0$ .

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## Poor Arguments $\longrightarrow$ Poor Proofs (2 / 2)

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**Conjecture:** For all  $n \in \mathbb{Z}^{\text{odd}}$ ,  $(n^2 - 1) \% 4 = 0$ .

Proof or Goof?:

Let  $x = 1$ .  $1^2 - 1 = 0$ .  $0 \% 4 = 0$ . Let  $x = 3$ .  $3^2 - 1 = 8$ .  
 $8 \% 4 = 0$ . Let  $x = 5$ .  $5^2 - 1 = 24$ .  $24 \% 4 = 0$ . This  
shows no sign of failing to give a result of 0.

Therefore, for all  $n \in \mathbb{Z}^{\text{odd}}$ ,  $(n^2 - 1) \% 4 = 0$ .

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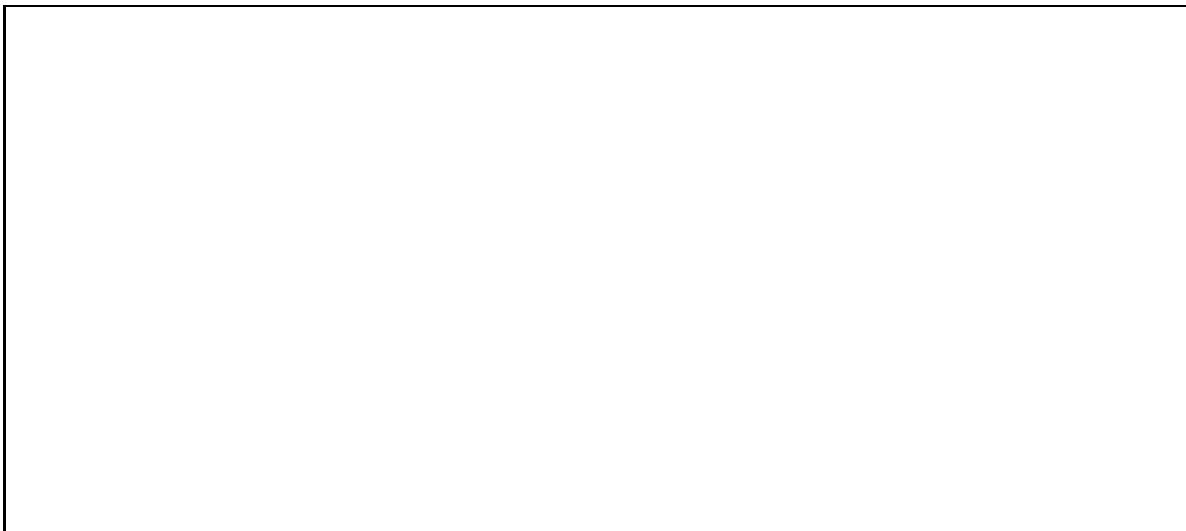
## Disproving Conjectures

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Typical approaches:

- (1) Prove that the conjecture's negation is true.
- (2) Find a counter-example. (Very commonly used!)

**Example(s):**



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