

# Topic 6:

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## Additional Set Concepts

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## Set Concepts Already Covered

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You may wish to review these basic set concepts, previously covered in the Math Review appendix, before starting this topic:

- Properties of sets (e.g., duplicate members are not allowed)
- Set notation (membership, set builder notation, etc.)
- Operators (union, intersection, difference, complement, cardinality)
- Venn diagrams

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# Why Are We Learning More About Sets?

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Sets are foundational in many areas of Computer Science.

For example:

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## Subsets

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### Definition: Subset

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### Definition: Proper Subset

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### Example(s):

# Set Equality

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## Definition: Set Equality

## Example(s):

# Power Sets

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## Definition: Power Set

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## Example(s):

# Generalized Forms of $\cup$ and $\cap$

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Remember summation and product notations? E.g.:

$$\sum_{n=0}^9 (2n + 1)$$

Similar notation is used to generalize the union and intersection operators.

Assuming that  $A_1 \dots A_m$  and  $B_1 \dots B_n$  are sets, then:

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## Two More Set Properties

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### Definition: Disjoint

### Definition: Partition

### Example(s):

# Examples of Set Identities

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Look familiar?

**Associativity**  $(A \cap B) \cap C = A \cap (B \cap C)$   
 $(A \cup B) \cup C = A \cup (B \cup C)$

**Commutativity**  $A \cap B = B \cap A$   
 $A \cup B = B \cup A$

**Distributivity**  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$   
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

**De Morgan**  $\overline{A \cup B} = \overline{A} \cap \overline{B}$   
 $\overline{A \cap B} = \overline{A} \cup \overline{B}$

**Note:** As with logical identities, you need not memorize set identities.

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## Set Builder Notation

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Often, set contents are easier to describe than to list:

Describe:  $10 \leq x \leq 99, x \in \mathbb{Z}$

List: 10, 11, 12, 13, ..., 98, 99

To describe a set's content, we use *set builder notation*:

Form:  $\{n \mid \text{description of the legal values of } n\}$

**Example(s):**

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# Expressing Set Operations in Logic

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We've seen the first two already.

$$X \subseteq Y \equiv \forall z (z \in X \rightarrow z \in Y)$$

$$X \subset Y \equiv \forall z (z \in X \rightarrow z \in Y) \wedge \exists w (w \notin X \wedge w \in Y)$$

For those that return sets, Set Builder notation is a good choice:

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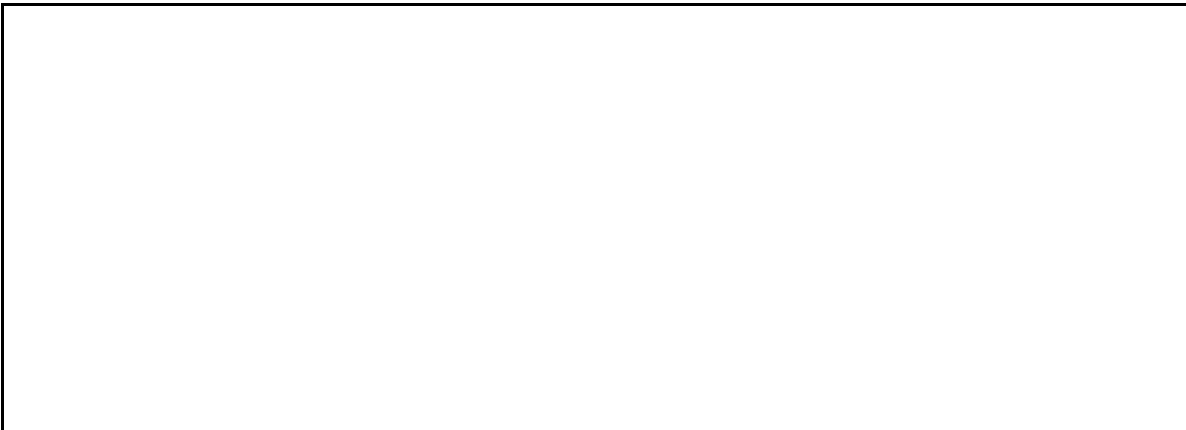
## Proving Set Identities (1 / 4)

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To prove that set expressions  $S$  and  $T$  are equal, we can:

1. Prove that  $S \subseteq T$  and  $T \subseteq S$ , or
2. Convert the equality to logic, prove it, and convert back

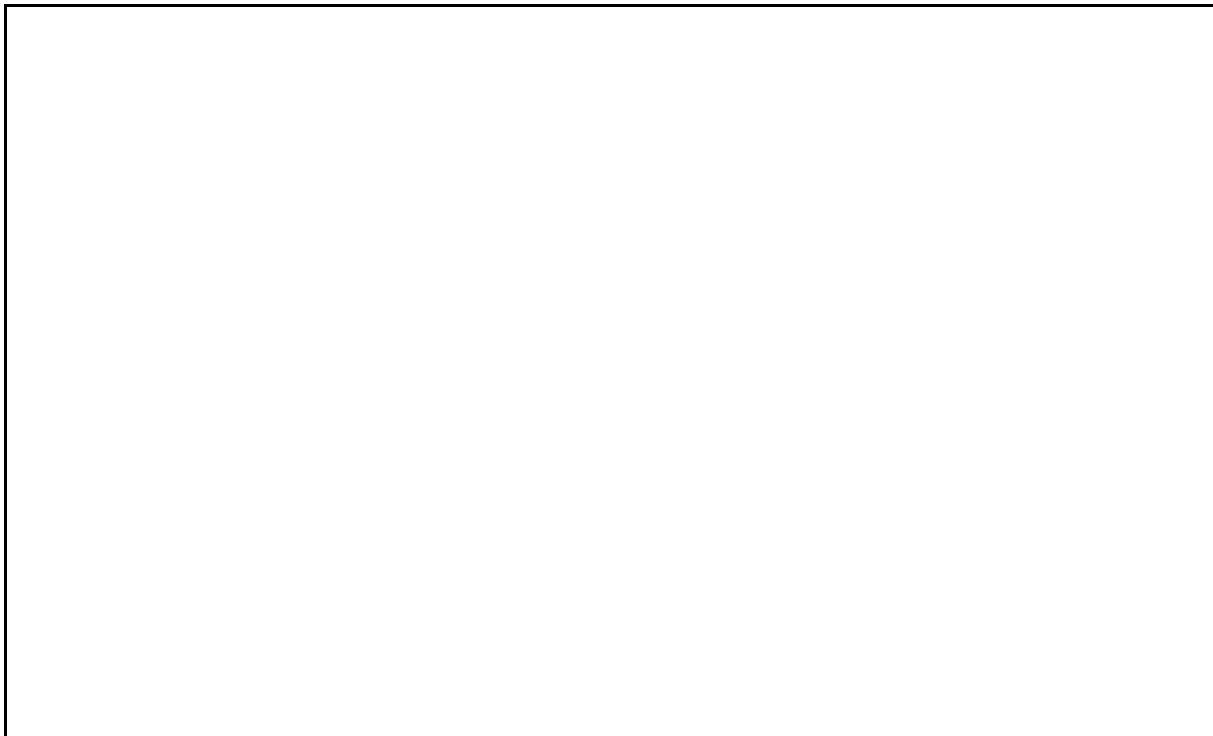
**Example(s):**



## Proving Set Identities (2 / 4)

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**Conjecture:**  $S \cup \mathcal{U} = \mathcal{U}$



## Proving Set Identities (3 / 4)

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## Proving Set Identities (4 / 4)

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**Conjecture:**  $S \cup \mathcal{U} = \mathcal{U}$

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## Final Set Operator: Cartesian Product (1 / 2)

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**Definition: Ordered Pair**

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**Example(s):**

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# Final Set Operator: Cartesian Product (2 / 2)

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## Definition: Cartesian Product

## Example(s):

Notes:

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# Example: Computer Representation of Sets

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