# Topic 8:

Relations



# Background

Having collections of data: Good.

Knowing the connections between collections: Better!

**Example(s):**

# Relations (1 / 2)

**Definition: (Binary) Relation**

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

**Example(s):**



# Relations (2 / 2)

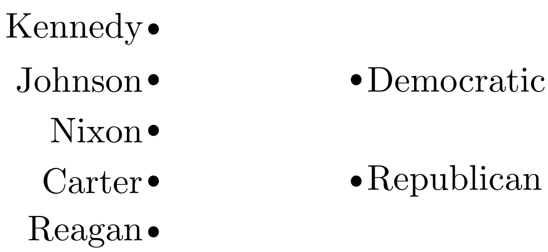
**Definition: Related**

**Example(s):**

# Graph Representations of Relations (1 / 2)

Example #1: Presidents–Parties

Recall: $A = \{$Kennedy, Johnson, Nixon, Carter, Reagan$\}$
$B = \{$Dem, Rep$\}$
$R = \{$(Kennedy, Dem), (Johnson, Dem), (Nixon,Rep), (Carter, Dem), (Reagan, Rep) $\}$

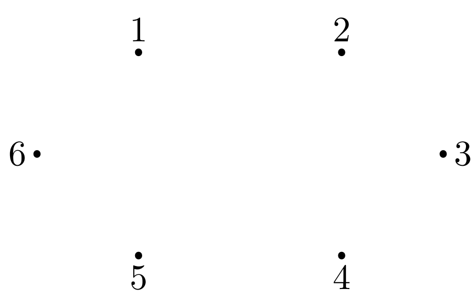





# Graph Representations of Relations (2 / 2)

Example #2: $x \,\%\, y = 0, x \neq y$

Recall: $H = \{1, 2, 3, 4, 5, 6\}$
$R = \{(2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (4, 2), (6, 2), (6, 3)\}$



# Properties of Relations: Reflexivity

**Definition: Reflexivity**

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**Example(s):**



# Properties of Relations: Symmetry (1 / 2)

**Definition: Symmetry**

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**Example(s):**

# Properties of Relations: Symmetry (2 / 2)

**Example(s):** Graph Representations & Symmetry



# Properties of Relations: Antisymmetry (1 / 2)

**Definition: Antisymmetry**

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**Example(s):**

# Properties of Relations: Antisymmetry (2 / 2)

**Example(s):** Graph Representations & Antisymmetry



# Properties of Relations: Transitivity (1 / 2)

**Definition: Transitivity**

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**Example(s):**

# Properties of Relations: Transitivity (2 / 2)

**Example(s):**



# Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators

# Relational Composition Examples (2 / 4)

Example #2: Swapping content of ordered pairs

**Definition: Inverse**



# Relational Composition Examples (3 / 4)

Example #3: Composites

**Definition: Composite**

**Example(s):**

# Relational Composition Examples (4 / 4)

Example #3: Composites (cont.)

**Example(s):**

**Definition: Complement**



# Matrix Representation of Relations (1 / 4)

*(Assumption: Relations are on just one set.)*

The 0-1 matrix representation of relation $R$ on set $A$ is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then `matrix[a][b]=1`. Else, `matrix[a][b]=0`.

**Example(s):**

# Matrix Representation of Relations (2 / 4)

Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1's along the main diagonal

**Example(s):**



# Matrix Representation of Relations (3 / 4)

Observation #2: Detecting Symmetry

⇒ Let matrix $M$ represent relation $R$. $R$ is symmetric when $m_{ij} = 1$ *iff* $m_{ji} = 1$ is true

**Example(s):**

# Matrix Representation of Relations (4 / 4)

Observation #3: Detecting Transitivity

⇒ Let matrix $M$ represent relation $R$. $R$ is transitive when no zero in $M$ becomes non–zero in $M^2$ (or in $M^{[2]}$).

**Example(s):**



# Equivalence Relations (1 / 4)

You may have already implemented one in Java...

**Definition: Equivalence Relation**

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# Equivalence Relations (2 / 4)

**Example(s):**



# Equivalence Relations (3 / 4)

So . . . why are these called *equivalence* relations?

Recall:

$$R = \{\ (0,0),\\ (1,1),(1,-1),(-1,1),(-1,-1),\\ (2,2),(2,-2),(-2,2),(-2,-2)\ \}$$

# Equivalence Relations (4 / 4)

**Definition: Equivalence Class**

**Example(s):**



# Partial Orders (1 / 3)

Consider scheduling the construction of a house.

**Definition: Reflexive (a.k.a. Weak) Partial Order**

# Partial Orders (2 / 3)

**Example(s):**



# Partial Orders (3 / 3)

**Definition: Irreflexivity (of Relations)**

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**Definition: Irreflexive (a.k.a. Strict) Partial Order**

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# Total Orders (1 / 2)

**Definition: Comparable**

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**Definition: Total Order**

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# Total Orders (2 / 2)

**Example(s):**