Homework #1
(50 points)

Due Date: January 26th, 2024, at the beginning of class

Directions
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1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**

2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via `piazza.com` and our office hours.

3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.

4. When your PDF is ready to be turned in, do so on `gradescope.com`. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on `https://help.gradescope.com/`.

5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Math Review

1. (6 points) For each of the following claims, answer 'True' or 'False.' If your answer is 'False,' provide an example that demonstrates that the claim is, indeed, false.

   (a) Multiplication is commutative on $\mathbb{R}$.

   (b) Division is associative on $\mathbb{R}$.

   (c) Subtraction is right–distributive over addition on $\mathbb{Z}$.

   (d) The relationship "is at least as old than," as applied to people, is transitive.

2. (3 points) For each of the following expressions, what is the most precise relationship that exists between m and q? Assume that m through q are all integers. If no clear relationship exists, explain why not.

   (a) $m \leq n < o \leq p \leq q$ (b) $o < q \leq p \geq m > n$ (c) $m \geq n \neq o \geq p > q$

3. (4 points) For each of the following decimal representations, determine the reduced fraction that represents the same value.

   (a) $12.\overline{10}$ (b) $8.84\overline{91}$

4. (6 points) If $\mathcal{U} = \{a, b, c, d, e, f, g, h, i, j\}, K = \{b, d, f, h, j\}, L = \{a, e, f, g, i, j\}$, and $M = \{b, f, g, h, i\}$, what is the result of the evaluation of each of the following set expressions? (Recall: $\mathcal{U}$ is the universe.)

   (a) $K - \mathcal{U}$ (c) $(K \cup M) \cap (L \cup M)$

   (b) $K \cap L \cap M$ (d) $(M - K) \cup \overline{L}$

5. (5 points) Evaluate each of the following expressions, producing a single integer result.

(a) $\displaystyle\sum_{k=2}^{5} (6 \% k)$

(b) $\displaystyle\sum_{j=-2}^{2} \prod_{k=3}^{5} (k - j)$

6. (4 points) Evaluate each of the following expressions, producing a single result value for each.

(a) $77 / 13$

(b) $77 \setminus 13$

(c) $77 \% 13$

(d) $77 \mid 13$

7. (4 points) Consider the expression $56 \equiv r \pmod{15}$. What are four values of r that make the expression true?

8. (6 points) Evaluate each of the following expressions, producing as reduced a result as possible. Assume all values are such that division by zero is not possible.

(a) $\dfrac{1}{1 - \frac{1}{1-h}}$

(b) $\dfrac{\displaystyle\prod_{b=2}^{5} b^2}{\displaystyle\prod_{a=1}^{4} a(a+1)}$

(c) $\dfrac{\frac{a}{b} \cdot \frac{b^2 c}{a^2 d}}{\frac{bc^2 d}{cd^2} + \frac{(ad)^2 - a^2 d^2}{d}}$

9. (8 points) Using the laws of exponents and logarithms presented in class, evaluate each of the following expressions to a numeric answer. (Remember, you won't have a calculator to use during quizzes and exams; now is the time to learn those laws and practice their application!)

(a) $\left(\dfrac{4^2}{(2^2)^3}\right)^2$

(b) $\dfrac{(au)^d}{a^{d-1}(au)u^{d-1}}$

(c) $\dfrac{\log_{13} 64}{\log_{13} 4}$

(d) $\log_5 \left(\dfrac{25^4}{5^6}\right)$

Topic: Propositional Logic

10. (4 points) For each of the following expressions, is it a proposition?

(a) $a > 7 > c$

(b) Comb your hair!

(c) Your shoelaces are untied.

(d) Horace's bike weighs 16 pounds.