CSc 245 — Introduction to Discrete Structures
Spring 2008 (McCann)

Collected Definitions for Exam #1

I can't recall the last time I didn't ask a definition question on an exam. To help you better prepare yourself for such questions, I've assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won't specifically ask you for their definitions on the exam.

Once in a while a student will express disappointment that I ask definition questions on exams. My justification is that I think it's important for you to know what the core terms mean so that you can use them correctly and effectively. At the same time, I don't require that you memorize the exact wording of the definitions you see here. If you provide a definition in your own words that captures all of the detail found here, that's fine.

The definitions are grouped by lecture topic, and should be in an order within each topic that is at least close to the order in which the definitions appeared in class.

Topic 1: Course Background

- *Discrete Mathematics* encompasses the representation and study of collections of distinct objects.

Topic 2: Logic

- *Philosophical Logic* is the classical notion of 'logic:' The study of thought and reasoning.
- *Mathematical Logic* is the use of formal languages and grammars to represent the syntax and semantics of computation.
- A *Well-Formed Formula (wff)* is a correctly structured expression of a language.
- A *proposition* (a.k.a. *statement*) is a claim that is either true or false with respect to an associated context.
- A *simple proposition* is a proposition that contains no logical operators.
- A claim that is a combination of multiple propositions is a *compound proposition.*
- Two propositions p and q are *(Logically) Equivalent* ($p \equiv q$) when both evaluate to the same result when presented with the same input. [Note: An alternate, equally-correct definition is given below.]
- A *Tautology* is a proposition that always evaluates to true.
- A *Contradiction* is a proposition that always evaluates to false.
- A *Contingency* is a proposition that is neither a tautology nor a contradiction.
- The *Inverse* of $p \rightarrow q$ is $\overline{p} \rightarrow \overline{q}$.
- The *Converse* of $p \rightarrow q$ is $q \rightarrow p$.
- The *Contraposition* of $p \rightarrow q$ is $\overline{q} \rightarrow \overline{p}$.
- p and q are *(Logically) Equivalent*, written $p \equiv q$, if $p \leftrightarrow q$ is a tautology. [Note: An alternate, equally-correct definition is given above.]

Topic 3: Quantification

- A statement that includes one or more variables and will evaluate to either true or false when the variables are assigned values is a *Predicate* (a.k.a. *Propositional Function*).
- The collection of values from which a variable's value is drawn is known as the *Domain of Discourse* (a.k.a. *Universe of Discourse*).
- A quantified variable in a predicate is a *Bound* variable.
- Unquantified variables are *Free* (a.k.a. *Unbound*) variables.

Topic 4: Arguments

- "An *Argument* is a connected series of statements to establish a definite proposition." [Thanks to Monty Python!]
- An argument that moves from specific observations to a general conclusion is an *Inductive Argument*.
- An argument that uses accepted general principles to explain a specific situation is a *Deductive Argument*.
- Any deductive argument of the form $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is *Valid* if the conclusion must follow from the hypotheses.
- A valid argument that also has true premises is a *Sound* argument.
- Any unsupported or improperly constructed argument demonstrates *Specious Reasoning*.
- A *Fallacy* is an argument constructed with an improper inference.

Topic 5: Proofs of $p \rightarrow q$

- A *Conjecture* is a statement with an unknown truth value.
- A *Theorem* is a conjecture that has been shown to be true.
- A sound argument that establishes the truth of a theorem is a *Proof*.
- A *Lemma* is a simple theorem whose truth is used to construct more complex theorems.
- A *Corollary* is a theorem whose truth follows directly from another theorem.