CSc 345 — Analysis of Discrete Structures
Spring 2008 (McCann)

http://www.cs.arizona.edu/classes/cs345/spring08/

Homework #1

(50 points)

Due Date: January 24^{th}, 2008, at the beginning of class

As we'll be doing plenty of work with proofs in this class, here's a homework assignment to help you awaken your dormant proofing skills. Most of the problems are straight–forward, but one or two might challenge you. Remember that $\mathbb{Z}$ is the set of all integers, $\mathbb{Z}^+$ is the set of all positive integers, and $\binom{n}{k}$ is combination notation (n choose k).

Write complete, legible answers to each of the following questions. Show your work, when appropriate, for possible partial credit. This is not a group project; do your own work. The TAs will go over the homework problems with you during the section meeting following the due date. *If you need help, remember that the TAs and I have office hours for just this eventuality, and you may post general questions to the class newsgroup.*

On the due date, at the start of class, turn in your neatly written (or, better yet, electronically–formatted) answers and turn them in to the TAs. Solutions submitted after the first five minutes of class on the due date will not be accepted. Want to be safe and submit your homework early? Feel free to do so; you can give it to me or to either TA during our office hours. I strongly encourage you to retain a copy of your submitted answers, just in case you need to refer to them before you get your graded homework back.

1. (6 points) Prove (by your choice of valid technique) or disprove: $\sum_{i=1}^{n} i = \binom{n+1}{2}$.
2. (6 points) Prove (by your choice of valid technique) or disprove: $(-1)^n = -1$ for any odd integer n.
3. (4 points) What is wrong with this following "proof?"

   Conjecture 1 *The product of an even integer with an odd integer is even.*

   Proof (Direct): *Let a be an even integer and let b be an odd integer. Because a is even, $a = 2p$ for some integer p. Because b is odd, $b = 2q + 1$ for some integer q. If ab is even, then there exists an integer r such that $ab = 2r$. Thus, $ab = (2p)(2q + 1) = 2r$. As ab is equal to twice r, ab must be even.*

   Therefore, the product of an even integer with an odd integer is even. $\diamondsuit$

4. (8 points) Prove (by your choice of valid techniques) or disprove: $\forall\, m, n \in \mathbb{Z}$, if $m + n$ is even then either m and n are both even or they are both odd.
5. (8 points) Prove (by contradiction) or disprove: Every integer > 11 is the sum of two composite integers. (A composite integer has a prime divisor greater than 1 but less than itself.)
6. (8 points) Prove (by induction) or disprove: $2n + 1 < 2^n, n \in \mathbb{Z}, \forall n \geq 2$.
7. (10 points) Consider the sequence defined by the rule $2k - 1, k \in \mathbb{Z}^+$.

   (a) Find a simple formula for $\sum_{k=1}^{n} (2k - 1)$.

   (b) Prove (inductively) that your simple formula is correct.