1 Recursion Trees

Draw a recursion tree for the recurrence

\[ T(n) = 3T\left(\frac{3n}{4}\right) + cn \]

Draw it at least to the third layer (that is, the grandchildren of the root); each node in the tree should show the local cost of that particular part of the recursion (see Slide Deck 04, slide 44).

Then make an argument (not a formal proof) that the tree will eventually have \( \Theta(\log n) \) layers, and that it will have \( \Theta\left(3^{\log_4 3} n\right) \) leaves.

(My apologies to those who print out their homework; I know that this part will be hard to do on a computer. Draw it by hand, or draw a picture in a Paint program. Or, if you really want to go crazy, write a .dot file to draw it for you.)

2 Master Method

Solve the following recurrences with the Master Method, if possible. Be clear to show the value of the constants \( a, b \). Also identify exactly which case you are using. If a logarithm can be easily simplified (such as \( \log_2 4 = 2 \)), do so; if not (such as \( \log_3 7 \)), you may either convert it to a decimal value, or keep it in logarithm form.

If the recurrence cannot be solved by the Master Method, state why.

(a)

\[ T(n) = 4T\left(\frac{n}{2}\right) + n \]

(b)

\[ T(n) = 3T\left(\frac{n}{3}\right) + n^3 \]

(c)

\[ T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n} \]

(d)

\[ T(n) = 4T\left(\frac{9n}{10}\right) + n^2 \]

(e)

\[ T(n) = 4T\left(\frac{10n}{9}\right) + n^2 \]
(f) 
\[ T(n) = 2T\left(\frac{n}{2}\right) + n\lg n \]

(g) 
\[ T(n) = 7T\left(\frac{n}{8}\right) + n^2\lg n \]

(h) 
\[ T(n) = 8T\left(\frac{n}{2}\right) + n^2\lg n \]

(i) 
\[ T(n) = 2T\left(\frac{n}{2}\right) + \lg n \]