Homework 1 Solutions

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1 Problem 1

If a computer could represent all real numbers, then function will never terminate. If there exists such a number x such that $x/2 = 0$, then that implies that $0 * 2 = x$. But that number can only be 0, and therefore since the input is a non-zero number, the function will never terminate. Alternatively, it can be proved by arguing that a number exists between every two numbers on the real number line.

2 Problem 2

$$log_b a = \frac{log_{10} a}{log_{10} b} \quad (0)$$

Assume that $log_b a = y$, then by raising both sides with a base of b, we get

$$a = b^y \quad (0)$$

Now take the log_{10} of both sides

$$log_{10} a = y \cdot log_{10} b \quad (0)$$

$$\frac{log_{10} a}{log_{10} b} = y = log_b a \quad (0)$$

3 Problem 3

3.1 2.19a

Proof (Inductive):

Basis: n=1

$1^2 - 1 = 0$ and is therefore even.

Inductive: $n^2 - n$ is even $\rightarrow (n+1)^2 - (n+1)$ is even

$(n+1)^2 - (n+1) = n^2 + 2n + 1 - n - 1 = (n^2 - n) + 2n$

From our hypothesis, we know that $n^2 - n$ is even. And since $2n$ is even and addition of even numbers is even, $(n+1)^2 - (n+1)$ is even.

QED

3.2 2.19b

Proof (deductive):

First, without loss of generality, we assume that n is even.

Then, $n^2 - n = n(n-1)$. Now, since $n-1$ is odd and even multiplied by odd is even, $n^2 - n$ is always even.

QED

3.3 2.19c

Proof (deductive):

By definition, every 3rd number is divisible by 3.

$n^3 - n = n(n^2 - 1) = n(n-1)(n+1) = (n-1)n(n+1)$

Given any value n, either $n-1$, n, or $n+1$ is a multiple of 3.

QED

3.4 2.19d

Proof (Inductive):

Basis: n=1

$1^5 - 1 = 0$ and is divisible by 5.

Inductive: $n^5 - n$ is even $\rightarrow (n+1)^5 - (n+1)$ is even

$(n+1)^5 - (n+1) = n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1 - n - 1 = (n^5 - n) + 5(n^4 + 2n^3 + 2n^2 + n)$

From our hypothesis, we know that $n^5 - n$ is even. The second term in parenthesis is a also divisible by 5, making the whole expression divisible by 5.

QED

4 Problem 4

Let $\sqrt{3} = \frac{a}{b}$, then $3 = \frac{a^2}{b^2}$ or $a^2 = 3b^2$ [1]. We now consider the following cases.

case 1: b is even. In this case b^2 is also even. Since $a^2 = 3b^2$, we can conclude that a^2 is even, and hence a is even. Since a and b can be reduced by canceling a common factor of 2, we have a contradiction.

case 2: b is odd. In this case b^2 is also odd. Since $a^2 = 3b^2$, we can conclude that a^2 is odd, and hence a is odd. Let $a = 2m + 1$ and $b = 2n + 1$. Substituting in the equation [1] we get $3(4n^2 + 4n + 1) = 4m^2 + 4m + 1$, or $6n^2 + 6n + 1 = 2(m^2 + m)$. Since the left hand side is an odd number and the right hand side is an even number we have a contradiction.

Hence $\sqrt{3}$ is not rational.

Here we show that a^2 and a are of same parity. To show this we consider $a^2 - a = a(a-1)$. Since either a or $a-1$ is even, we get $a^2 - a$ is even, which implies a^2 and a are of same parity.

5 Problem 5

If we represent every object the position on a bit vector and use 0's to represent absence and 1's to indicate presence, then `001010` would represent a subset of the power set containing only the 2nd and 4th element for a set containing 6 objects. Therefore, representing all possible subsets is the same as asking all possible permutations of the bit string, which is `2*2*2*2*2*2` $= 2^6$ for a set with 6 objects. Then, for any arbitrary n, the total number of subsets is then 2^n.

6 Problem 6

```
function reverse_list(Node head)
{
  if head == NULL
    return head
  else if head.next == NULL
    return head
  else
    Node cur  = head
    Node prev = cur
    Node next = cur.next
    cur.next = NULL
```

```
    while next != NULL
      prev = cur
      cur  = next
      next = cur.next
      cur.next = prev

    return cur
}
```