Homework 7

Due Monday, August 4, at 9 AM (GMT-7)

CSc 345 – Summer 2014

Instructor: Qiyam Tung

Instructions

1. This is an individual assignment. You must do your own work.
2. If you are having difficulty and need to ask a question you can:
   (a) Ask questions in class.
   (b) Stop by my office hours (or make an appointment).
   (c) Post a question on Piazza.
   (d) Post a private question on Piazza if the question is too specific.
3. Show all work. Incomplete solutions will **not** receive full credit
4. You may write your solutions by hand, or you may type them using any appropriate program such as Microsoft Word, OpenOffice Writer, LaTeX, etc...
   However, the final copy should be in PDF form and formatted so that it is legible.
5. If the listed problem is only a number, refer to the online book for the description of the problem (starting at page 46).

1 Problems (64 pts)

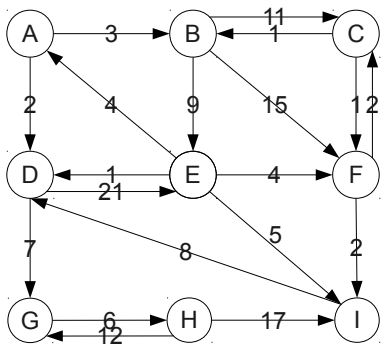

Figure 1: A weighted directed graph G

1. (8) Run Dijkstra's algorithm on the graph in Figure 1 for node A. Show your work, by showing all known path information at each step. Run the algorithm to completion.
2. (4) What is the **cost** of shortest path from A to I in the graph in Figure 1? **Draw the path** and **show how your table can be used to derive it**.
3. (3) Run BFS on Figure 1 starting at node A. Show the resulting tree. Label the edges in the order they were picked. You do not need to label the edge weights.
4. (3) Run DFS on Figure 1 starting at node A. Show the resulting tree. Label the edges in the order they were picked. You do not need to label the edge weights.
5. (4) Run Kruskal's MCST algorithm on the graph in Figure 2. Show the resulting tree. Label the edges in the order they were picked.
6. (4) Run Prim's MCST algorithm on the graph in Figure 2 starting at node A. Show the resulting tree. Label the edges in the order they were picked.

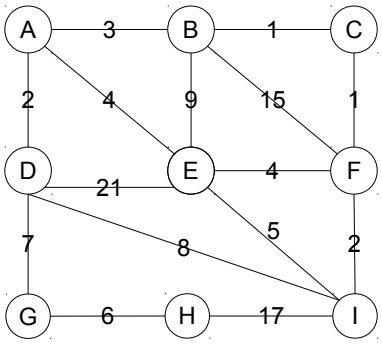

Figure 2: A weighted undirected graph G

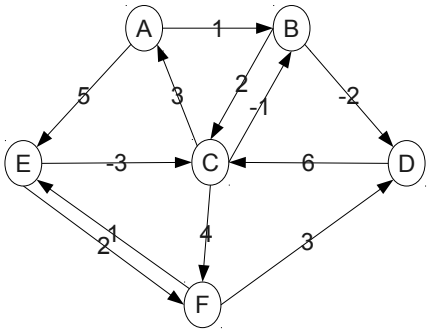

Figure 3: A weighted directed graph G

7. (8) Run Bellman-Ford algorithm on the graph in Figure 3 for node A. Show your work, by showing all the updates at each step. Run the algorithm to completion.

8. (2) Is there a shortest path from A to F in the graph in Figure 3? If so, draw the path, otherwise, explain.

9. (4) We know that Dijkstra's algorithm has trouble with graphs that have negative weight edges. Sometimes it gives a correct solution, sometimes it doesn't.

   (a) (2) Draw a graph with at least 3 vertices and with a negative weight edge for which Dijkstra's algorithm will produce a **correct** result.

   (b) (2) Draw a graph with at least 3 vertices and with a negative weight edge for which Dijkstra's algorithm will produce an **incorrect** result.

10. (8) Prove by induction that an undirected graph with n vertices has at most $\frac{n(n-1)}{2}$ edges.

11. (8) Suppose you are given a graph of a telephone network, which is a graph G whose vertices represent switching centers, and whose edges represent communication lines between two centers. The edges are marked by their bandwidth. The bandwidth of a path is the bandwidth of its lowest bandwidth edge. Give an algorithm that, given a graph and two switch centers a and b, will output the maximum bandwidth of a path between a and b. (hint: try to modify one of the graph algorithms we discussed in class)

12. (8) Prove the following implications.

    (a) (4) If an undirected graph is connected and has no simple cycles, then the graph has $|V| - 1$ edges.

    (b) (4) If an undirected graph has $|V| - 1$ edges and no cycles, then graph is connected.