Participation 7

Due Thursday, August 7, at 9 AM (GMT-7)

CSc 345 – Summer 2014

Instructor: Qiyam Tung

Instructions

1. This is an individual assignment. You must do your own work.
2. If you are having difficulty and need to ask a question you can:
   (a) Ask questions in class.
   (b) Stop by my office hours (or make an appointment).
   (c) Post a question on Piazza.
   (d) Post a private question on Piazza if the question is too specific.
3. Show all work. I will be grading on whether you put effort into this problem (i.e. participation) and not correctness. Showing your work helps me identify your thought process and helps me with grading.
4. You may write your solutions by hand, or you may type them using any appropriate program such as Microsoft Word, OpenOffice Writer, LaTeX, etc. . .
   However, the final copy should be in PDF form and formatted so that it is legible.

Universal Sets

Set theory is a useful foundation for most of the work that we do in computer science. For example, graphs can be represented as two sets and even ordered relations can be represented as sets (for example, $(1, 2) = \{\{1\}, \{1, 2\}\}$). However, there is a reason the set theory that is typically taught in undergraduate courses is often called naïve set theory.

Consider this: is there such a thing as a universal set? That is, does there exist a set that contains all possible sets?

Instead of tackling the question directly, it is easier to look at some of the possible sets that may exist in this universal set. In particular, we look at the set that contains all sets that do not contain themselves. This can be written formally as

$$A = \{x|x \notin x\}, \tag{1}$$

where x is a set.

Prove that this set cannot exist and therefore the universal set cannot exist either. *Hint: to prove this, determine whether $A \in A$.*