Point location and Persistent Data structures

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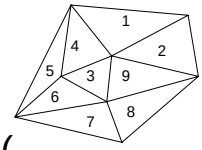
First problem

- Given a subdivision of the plane into triangles, create a data structure, such that given a query point, we can find in which triangle it lies.

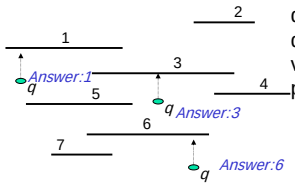


Too hard ;-(

A question that we would solve…

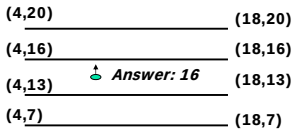
- A simpler problem

Given a set of horizontal segments, create a data structure, so that given a query point q, we can answer quickly which segment is vertically above the query point



A really really really easy question…

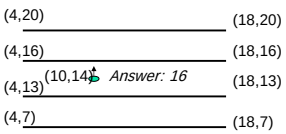
- Same question, but for a set of horizontal segments, all having the same x-coordinates.

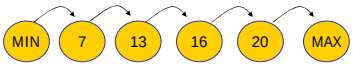


For simplicity, we use the y-coordinate of a segment as the "name" of the segment

Solution: Skip list

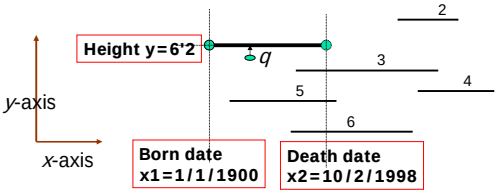
Solution: Store the y-coordinates of the segments in a SkipList.
Once a query point (x,y) is given, perform *succ(y)*





Different question

- In a city, people are born, and die.
- Each person is recognized by its height (no two people have the same height). We denote height by the letter *y*, and (birth/death) date by *x*.
- Need a DS that would find *Find(x,y)* - who was the person that was alive at date *x*, and her/his height is *y* (or if not exists), larger and as close as possible to *y*.



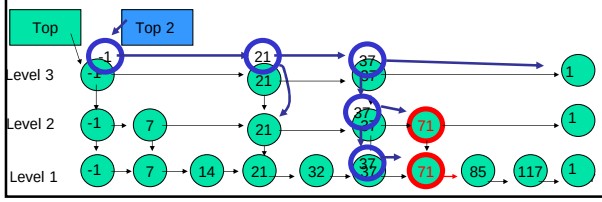
Different problem

How to delete an element from the skipList, without destroying it?

Assume we want to delete(71)

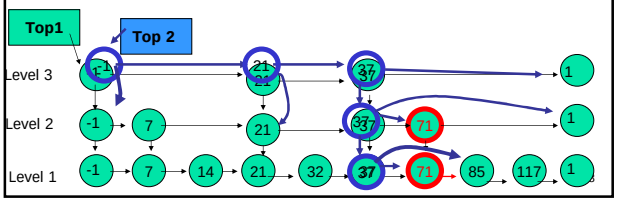
Idea #1: Copy the whole SkipList, and delete - too much memory

Idea #2: Copy the path that changes during the deletion, then modify this path.



Virtually copying SL

- To create the new virtual copy:
- Start from ***TOP*** of the old SL, create a new top, named ***Top2***
- Do ***find(x)*** /* ***x*** is the key to be deleted */
- Copy and connect every element along the search path.
- Delete ***x*** from the SL pointed to by ***Top2*** /*it does not affect the SL pointed to by ***Top1*** - only blue pointers change ***/**
- *Need to be a bit careful in the deletion (next slide)*



Need a little new function

- Follower(SL* sl, int x , int d){
  - // Returns a pointer to the smallest cell at level
  - // d, with key strickly greater than x
- P=sl->top; int d1=sl -> l ;
- while(1) {
  - while(p->key < x) p = p->nxt ;
  - if(d1 == d) return p ;
  - assert(p->down != NULL) ; //add #include<assert.h>
  - p = p -> down ;
  - d1-- ;
- }

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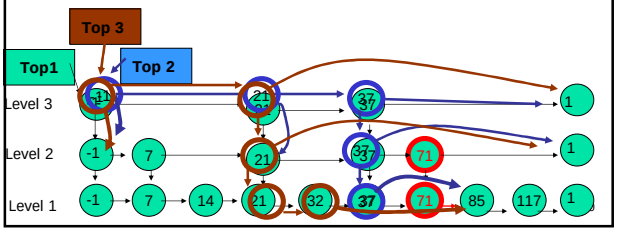
Virtually copying SL

To delete 37 -

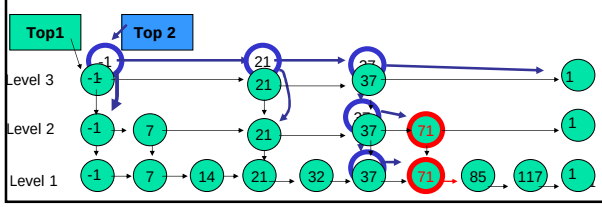
We copy as before the search path (brown path)

In each level d at which appear, we delete it using the command p->nxt = follower(sl, 37, 3)



Something fishy

- But it should not be.
- The new SL (lets call it SL(2)) is a perfectly legal SL. It has the same keys of SL(1), excluding 71 that was deleted.

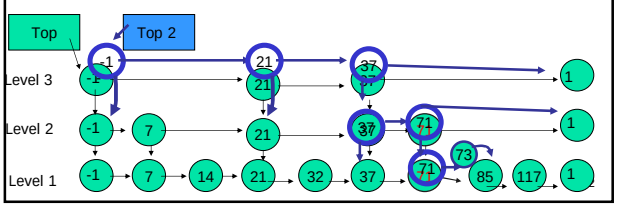


Inserting a key

How to insert a key into the skipList, without destroying it ?

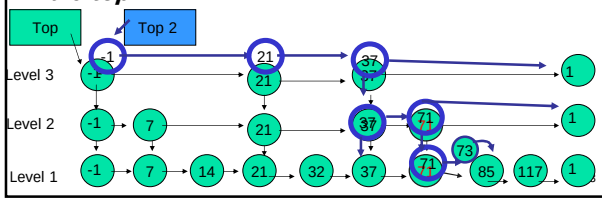
Same idea: Assume we want to ***insert(75)***

- Do ***search(75).***
- Copy every element that the search goes through.
- Let ***Top2*** point to the top of the list.
- ***Insert(72)*** into the SL pointed to by ***Top2 -*** only blue elements change



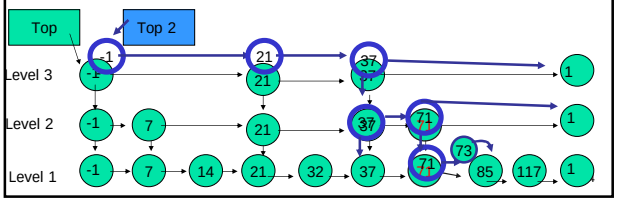
Again - a brandnew SL

- Note - again we obtained a perfectly legal SL.
- We have two SkipLists - one contains 73, the other one does not contain 73.
- We can now insert/delete elements into/from ***SL(2)***
- **Remember:** to access a SL, one only need the root - the ***top.***



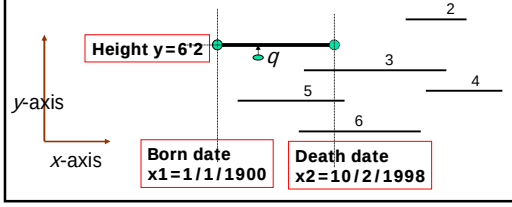
How much space

- **We saw that the average length of a path is O(log n), so each insert/delete takes O(log n) time and space**



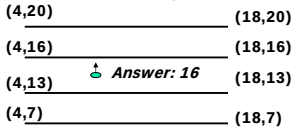
Back to the birth/death question

- In a city, people are born, and die.
- Each person is recognized by its height (no two people have the same height). We denote height by the letter *y*, and (birth/death) date by *x*.
- Need a DS that would find *Find(x,y)* - who was the person that was alive at date *x*, and her/his height is *y* (or if not exist), larger and as close as possible to *y*.

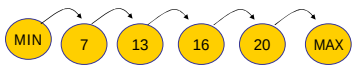


And remembering that this one is easy…

- All births/deaths start at the same date
- Call this problem the *same-population problem* (no births no deaths)

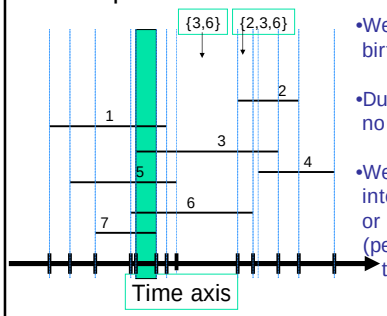


Easily solved via standard skip list



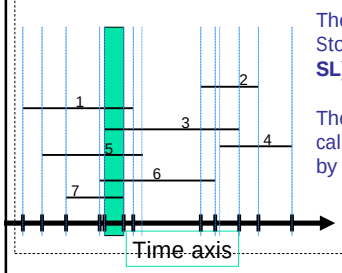
We can solve this one by…

- We split the time axis into *time-intervals.*



- We split the time at each birth or death.
- During the same interval, no birth or death occurs.
- We create the SL of one interval by inserting (birth) or deleting a segment (person) from the SL of the previous interval

How to access the different SL ?



So we obtain 2n+1 SkipList**s**, (one for time intervals).

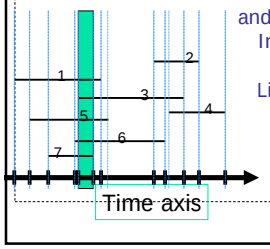
The roots of all different SL are Stored in a new SL (call it **dates SL**), sorted by date.

The SkipLists of the people are called **People SL,** sorted by height.

How to access the different SL ?

Alg: Create an empty people-SL
Create the dates-SL, and insert each birth/death event.

Scan these event (in increasing dates), and for each event (date) do
- Insert/delete (virtually) into/from the people SL.
- Link the TOP of the new people-SL with the currant key of the dates-SL



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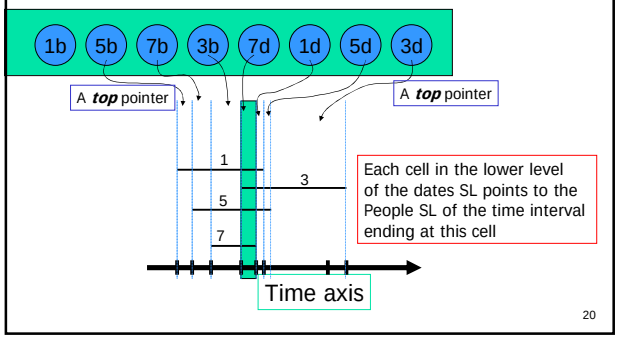
Overall data structure

- Dates SL (only lower level shown)



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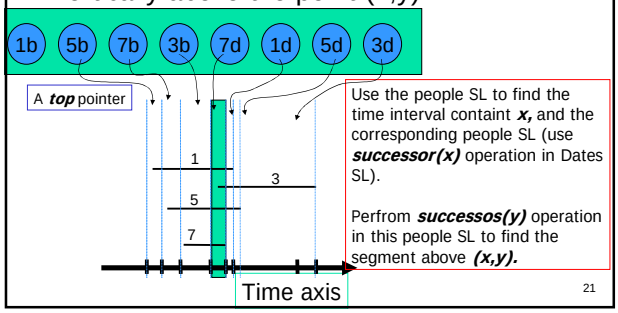
Answering a query

A query point (x,y) - which segment is vertically above the point (x,y)



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