Hidden Surface Removal

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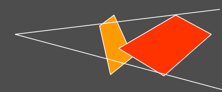
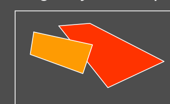
Hidden Surface Removal for Polygonal Scenes

- Input: Set of polygons in three-dimensional space + a viewpoint

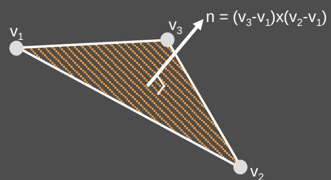


- **Output:** A two-dimensional image of projected polygons, containing only visible portions



2

The Normal Vector



n(1,2,3) = n(2,3,1) = -n(2,1,3)

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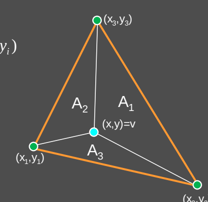
Barycentric Coordinates

$$(x,y)=\sum_{i=1}^{3}\alpha_i\cdot(x_i,y_i)$$

$$\alpha_i = A_i / \sum_{i=1}^{3} A_i$$

$$\sum_{i=1}^{3}\alpha_i = 1$$



Barycentric coordinates of v = (α_1, α_2, α_3)
B.C. are unique.
B.C. of all interior points are ≥ 0.
Triangle centroid = (1/3,1/3,1/3).

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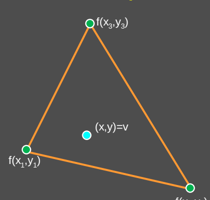
Linear Interpolation



$$(x,y)=\sum_{i=1}^{3}\alpha_i\cdot(x_i,y_i)$$

$$f(x,y)=\sum_{i=1}^{3}\alpha_i\cdot f(x_i,y_i)$$

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Back Face Culling (object space)

- In closed polyhedron you don't see object "back" faces
- Assumption
  - Normals of faces point *out* from the object
- Object space algorithm

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Back Face Culling

- Determine back & front faces using sign of inner product $< n, V >$

$$\langle n,v \rangle = n_x v_x + n_y v_y + n_z v_z = \|n\| \cdot \|v\| \cos\theta$$

- In a convex object :
  - Invisible back faces
  - All front faces entirely visible ⇒ solves hidden surfaces problem
- In non-convex object:
  - Invisible back faces
  - Front faces can be visible, invisible, or partially visible

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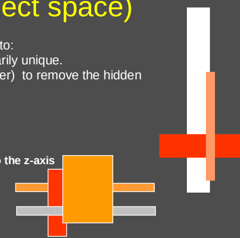
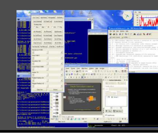
Depth Sort (object space)

- **Question:** Given a set of polygons, is it possible to:
  - sort them by depth. The order is not necessarily unique.
  - Then paint them back to front (over each other) to remove the hidden surfaces ?
  - This is called the **painter algorithm.**
  - **Note:**
    - **Each polygon is fully rendered,**
    - **polygons are not necessarily orthogonal to the z-axis**



- **Answer:** Usually not
- Works for special cases
  - E.g. polygons with constant z (where do we have polygons with constant z!?)



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Depth Sort (object space)

- Will fail for:
  - Intersecting polygons
  - Mutually occluding polygons

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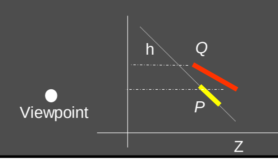
Plane containing *P*

- Since every polygon is planar, we can speak about the plane *h* of a polygon *P.*
- *Observation:* If polygon *Q* does not intersect *h*, then
  - If Q and the viewer lie on the same side of *h*, then during the painter algorithm, we can render *Q* before rendering *P*
  - Otherwise *P* can be rendered before *Q*



Depth Sort by Splitting

- Given two polygons, P and Q, we can order them in z if:
  1. P and Q do not overlap in their x extents
  2. Or P and Q do not overlap in their y extents
  3. Or P is totally on one side of Q's plane
  4. Or Q is totally on one side of P's plane
  5. Or P and Q do not intersect in projection plane
- Can we always resolve the relation between P and Q using steps 1-5?

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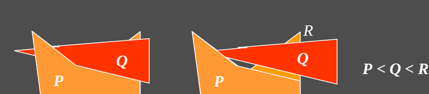
Depth Sort by Splitting

- What steps 1-5 all fail ?
- Split P into two smaller polygons using the plane h containing Q



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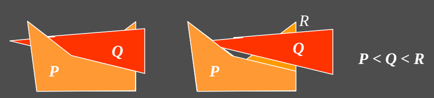
Painter's algorithm revised

- Given polygons $P_1 \ldots P_n$ in 3D,
- For each P_i, find h the plane containing P_i
  - For each other (sub)polygon Q, intersecting h
    - Split Q into two smaller polygons, one on each side of h.



- Looks like lots of unnecessary chopping…

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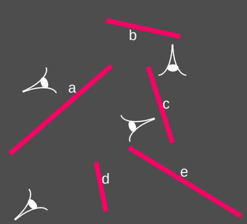
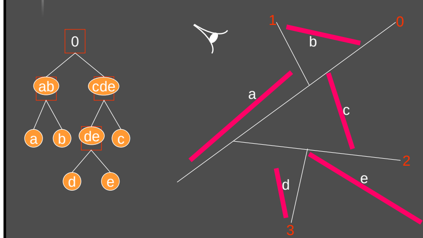
BSP - trees

- Construct a tree that gives a rendering order
- Tree recursively splits 3D world into cells, each of which contain at most one piece of polygon.
- Constructing tree:
  - choose polygon (arbitrary)
  - split its cell using plane on which polygon lies
  - continue until each cell contains only one polygon

The Visibility Problem



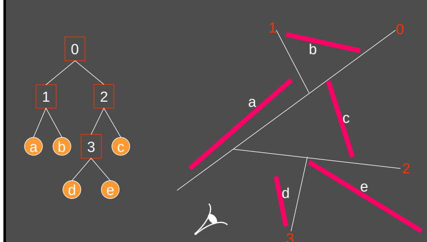
Binary Planar Partitions



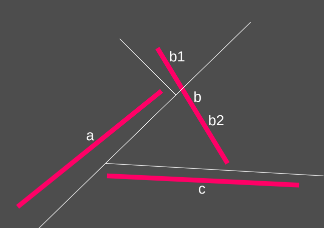
Draw everything below the line 0 (farther from the eye) before drawing the ones closer to the eye. Continue recursively inside each subtree.

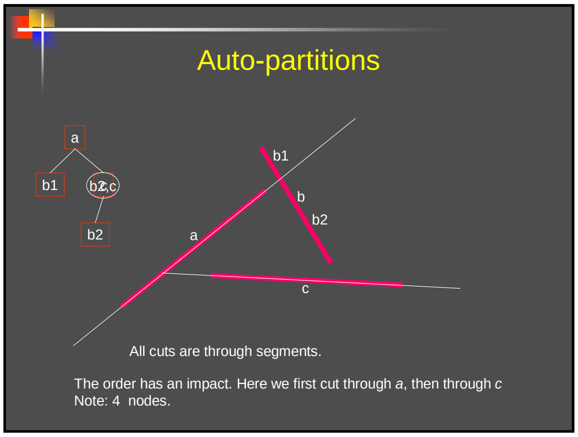
This structure, as any other hierarchical decomposition, is useful for range searching, point location etc.

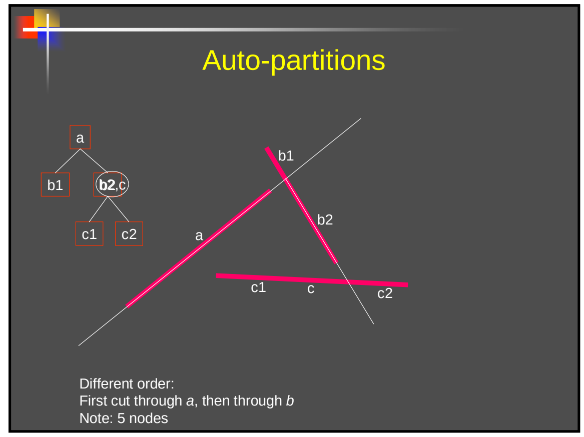
Painter's Algorithm

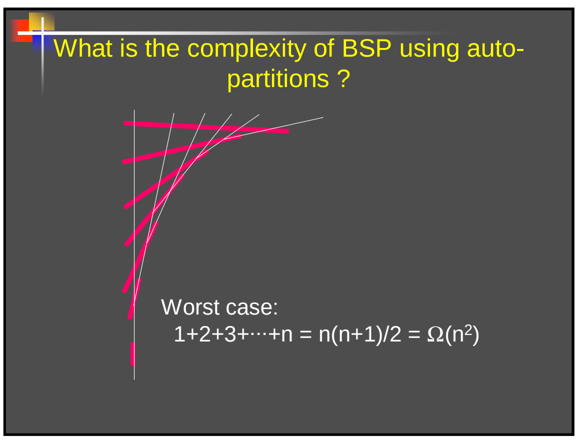


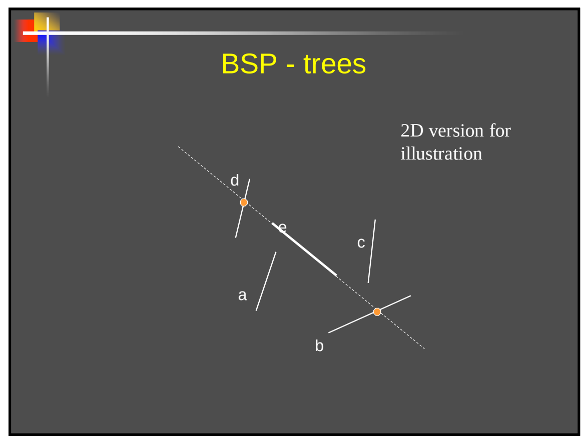
Binary Planar Partitions

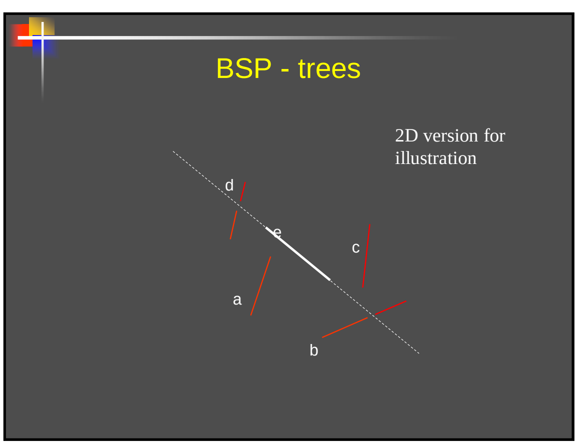


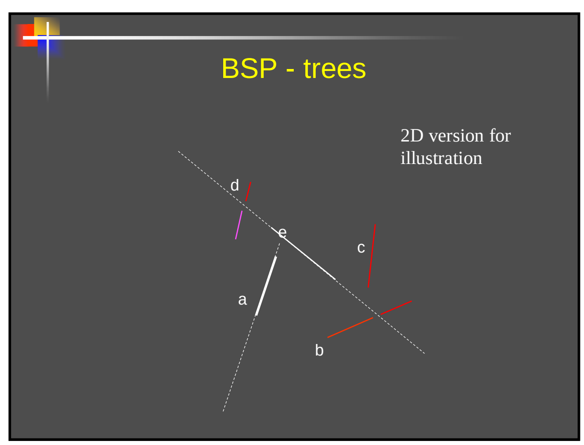

Auto-partitions
a
b1
b2,c
b2
b1
b
b2
a
c
All cuts are through segments.
The order has an impact. Here we first cut through *a*, then through *c*
Note: 4 nodes.


Auto-partitions
a
b1
b2,c
c1
c2
b1
b2
a
c1
c
c2
Different order:
First cut through *a*, then through *b*
Note: 5 nodes


What is the complexity of BSP using auto-partitions ?
Worst case:
1+2+3+···+n = n(n+1)/2 = Ω(n²)


BSP - trees
2D version for illustration
d
e
c
a
b


BSP - trees
2D version for illustration
d
e
c
a
b

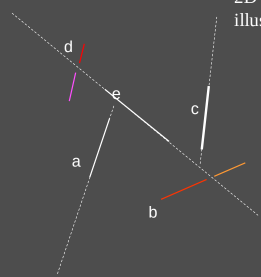

BSP - trees
2D version for illustration
d
e
c
a
b

Computer Graphics

Hidden Surface Removal

BSP - trees

2D version for illustration

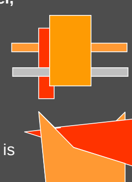


BSP - trees

- Rendering tree:
  - recursive descent
  - render back, node polygon, front
    - back/front is determined by what side of the plane the camera is on
- Disadvantages:
  - many small pieces of polygon (more splits than depth sort!)
  - over rendering (does not work well for complex scenes with lots of depth overlap)
- Advantages:
  - one tree works for all focal points (good for cases when scene is static)
  - filter anti-aliasing works fine, as does transparency
  - data structure is worth knowing about
- Comment
  - expensive to get approximately optimal tree, but for many applications this can be "off-line" in a pre-processing step.

Z-Buffer Algorithm (image space)

- **Basic Idea: resolve the visibility at the pixel level, using depth sort.**
- For each image pixel - store both the color and the current *z* depth
- Instead of always painting the pixels while scan-converting a polygon, do so only if polygon's depth is less than current *z* depth at that pixel



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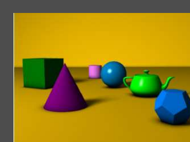

A simple three dimensional scene

Z-buffer representation

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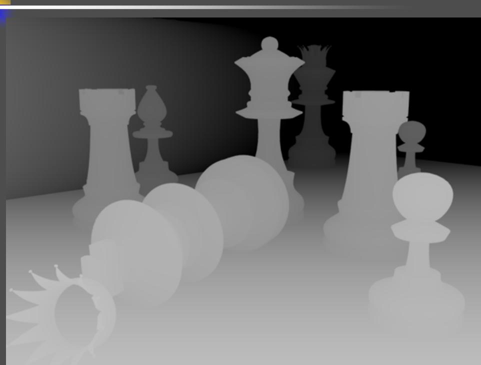

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Z-Buffer

```
ZBuffer(Scene)
For every pixel (x,y) do PutZ(x,y,MaxZ);
For each polygon P in Scene do
   Q := Project(P);
   For each pixel (x,y) in Q do
      z := Depth(Q,x,y);
      if (z < GetZ(x,y)) then
         PutZ(x,y,z);
         PutColor(x,y,Col(P));
      end;
   end;
end;
```

Questions: How can one compute **Project**(P) and **Depth**(Q,x,y)?

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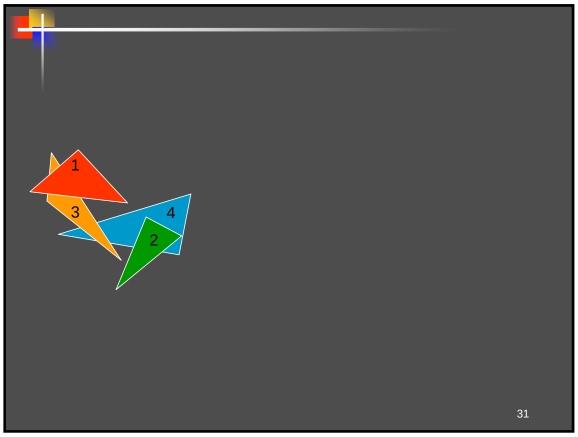
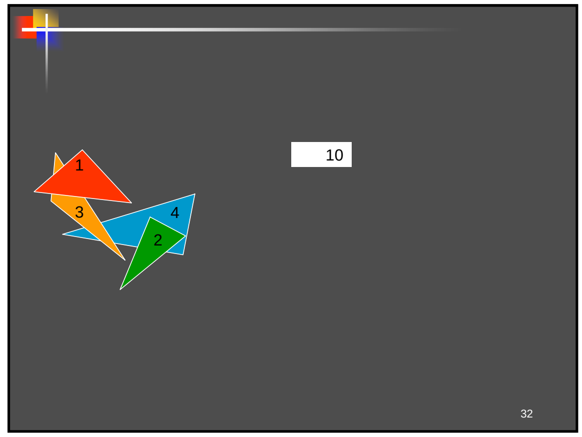
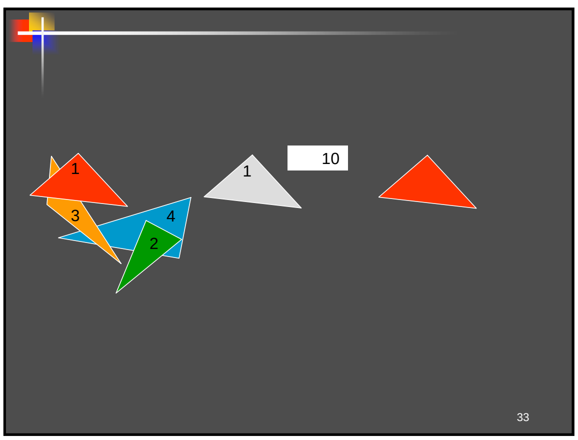
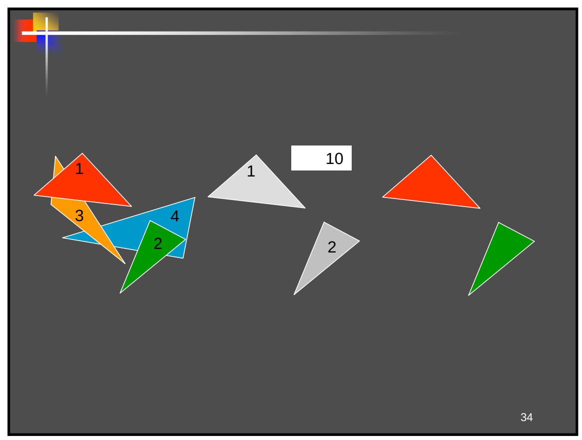
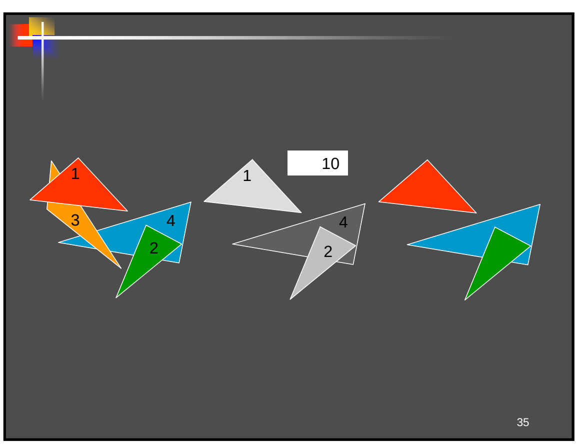
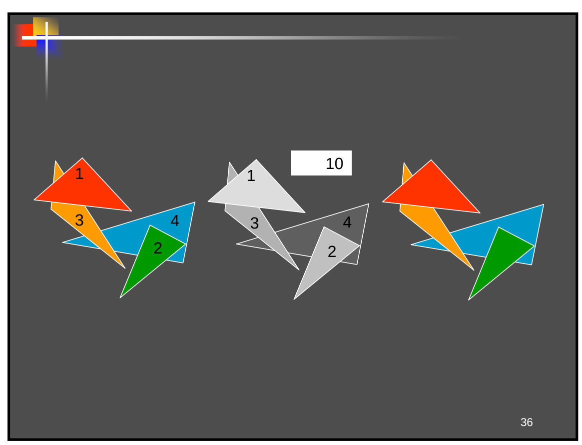

1
3
4
2
31


10
1
3
4
2
32


10
1
3
4
2
1
33


10
1
3
4
2
1
2
34


10
1
3
4
2
1
4
2
35

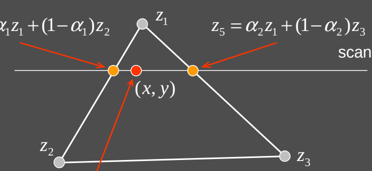

10
1
3
4
2
1
3
4
2
36

Z-Buffer – Depth(Q,x,y)

- In most cases, polygons are given by specifying their vertices.
- For the Z-buffer, we need to find the depth of two triangles in the same pixel.
- Linear interpolation will do.

$z_4 = \alpha_1 z_1 + (1-\alpha_1) z_2$

$z_5 = \alpha_2 z_1 + (1-\alpha_2) z_3$



$\text{Depth}(Q,x,y) = \alpha_3 z_4 + (1-\alpha_3) z_5$

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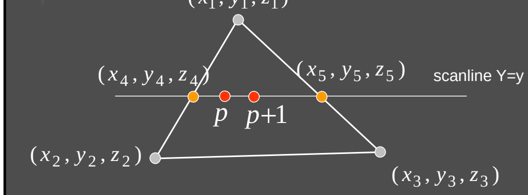
Z-Buffer – Depth(Q,x,y)



$$Z_p = \frac{x_5 - x_p}{x_5 - x_4} z_4 + \frac{x_4 - x_p}{x_5 - x_4} z_5$$

$$z_{p+1} = \frac{x_5 - x_p - 1}{x_5 - x_4} z_4 + \frac{x_4 - x_p + 1}{x_5 - x_4} z_5 = z_p + \frac{z_5 - z_4}{x_5 - x_4}$$

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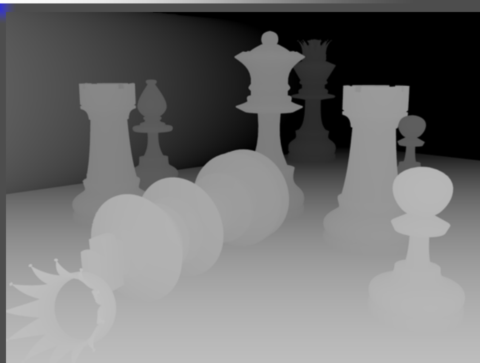

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Z-Buffer Algorithm

- Image space algorithm
- Data structure: Array of depth values
- Common in hardware due to simplicity
- Depth resolution of 32 bits is common
- ***Scene may be updated on the fly, adding new polygons***

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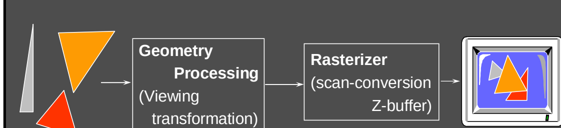
The Graphics Pipeline

- Hardware implementation of screen Z-buffer:
  - Polygons sent through pipeline one at a time
  - Display updated to reflect each new polygon



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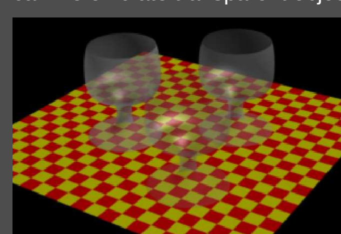
Transparency Z-Buffer

How can we emulate transparent objects?



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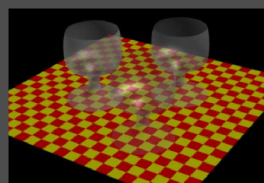
Transparency Buffer

- Extension to the basic Z-buffer algorithm
- Save *all* pixel values
- At the end – have list of polygons & depths (order) for each pixel
- Simulate transparency by weighting the different list elements, in order



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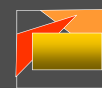
The A - buffer

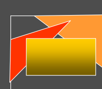
- For transparent surfaces and filter based anti-aliasing:
- Algorithm (1): filling buffer
  - at each pixel, maintain a pointer to a list of polygons sorted by depth.
  - when filling a pixel:
    - if polygon is opaque and covers pixel, insert into list, removing all polygons farther away
    - if polygon is opaque and only partially covers pixel, insert into list, but don't remove farther polygons



pixel

The A - buffer

- Algorithm (2): rendering pixels
  - at each pixel, traverse buffer using brightness values in polygons to fill.
  - values are used for either for calculations involving transparency or for filtering for aliasing



pixel

Scan-Line Z-Buffer Algorithm

- In software implementations - amount of memory required for screen Z-buffer may be prohibitive
- Scan-line Z-buffer algorithm:
  - Render the image one line at a time
  - Take into account only polygons affecting this line
- Combination of polygon scan-conversion & Z-buffer algorithms
- Only Z-buffer the size of scan-line is required.
- Entire scene must be available a-priori
- Image cannot be updated incrementally

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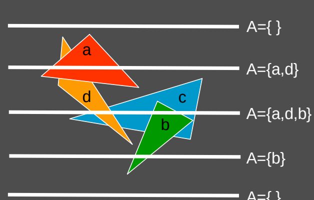

- The list *A* contains the polygons intersecting the horizontal sweeping line, in their order of appearance along the line changes only at vertices.

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Scan-Line Z-Buffer Algorithm

```
ScanLineZBuffer(Scene)
Scene2D := Project(Scene);
Sort Scene2D into buckets of polygons P in  increasing YMin(P) order;
A := EmptySet;
for y := YMin(Scene2D) to YMax(Scene2D) do
   for each pixel (x, y) in scanline Y=y do  PutZ(x, MaxZ);
   A := A + {P in Scene : YMin(P)<=y};
   A := A - {P in A : YMax(P)<y};
   for each polygon P in A
     for each pixel (x, y) in P's spans on the scanline
        z := Depth(P, x, y);
        if (z<GetZ(x)) then
          PutZ(x, z);
          PutColor(x, y, Col(P));
       end;
     end;
   end;
end;
```