

Computational Geometry

Chapter 7

Voronoi Diagrams

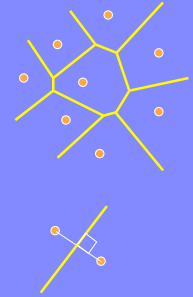
17.

Voronoi Diagram

- **Input:** A set of points locations (*sites*) in the plane.
- **Output:** A planar subdivision into cells. Each cell contains all the points for which a certain site is the closest.

Application: Nearest-neighbor queries (by point location in the diagram).

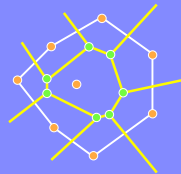
The *bisector* between two points is a line.



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Voronoi Diagram

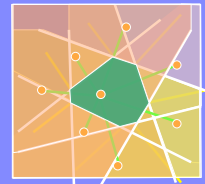
- ❑ Assume no four sites are co-circular.
- ❑ The Voronoi diagram is a planar graph, whose vertices are equidistant from three sites, and edges equidistant from two sites.
- ❑ The convex hull of the sites are those who have an unbounded cell. **Prove!**



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Voronoi Diagram – Naïve Construction

- ❑ Construct a bisector between one site and all others.
- ❑ A Voronoi cell is the intersection of all half-planes defined by the bisectors.
- ❑ Time complexity: $O(n \log n)$ for *each* cell.

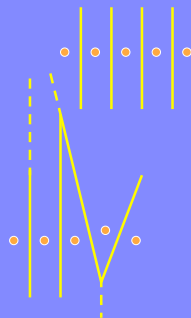


- ❑ **Corollary:** Each cell in a Voronoi diagram is a convex polygon, possibly unbounded.

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Voronoi Diagram

- ❑ If all the sites are colinear, the Voronoi diagram will look like this:
- ❑ Otherwise, the diagram is a connected planar graph, in which all the edges are line segments or rays

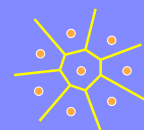


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Voronoi Diagram Complexity

- ❑ A Voronoi diagram of n distinct sites contains n cells.
- ❑ One cell can have complexity $n-1$, but not all the cells can.

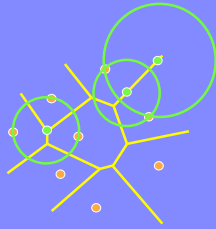
- The number of vertices $V \leq 2n-5$
- The number of edges $E \leq 3n-6$
- The number of faces $F = n$



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Voronoi Diagram Properties

- ❑ A vertex of a Voronoi diagram is the center of a circle passing through three sites.
- ❑ Each point on an edge is the center of a circle passing through two sites.



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