Tries and suffixes trees

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Trie: A data-structure for a set of words

All words over the alphabet Σ={a,b,..z}.
In the slides, let say that the alphabet is only {a,b,c,d}
S – set of words = {a,aba, a, aca, addd}
Need to support the operations
- insert(*w*) – add a new word *w* into *S*.
- delete(*w*) – delete the word *w* from *S*.
- find(*w*) is w in S ?
  - Future operation:
    - Given text (many words) where is *w* in the text.

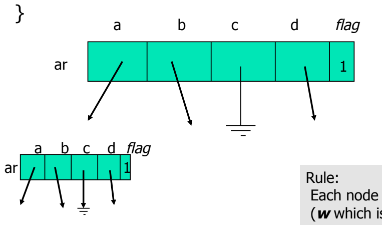
- The time for each operation should be O(*k*), where *k* is the number of letters in *w*

- Usually each word is associated with addition info – not discussed here.

Trie (Tree+Retrive) for *S*

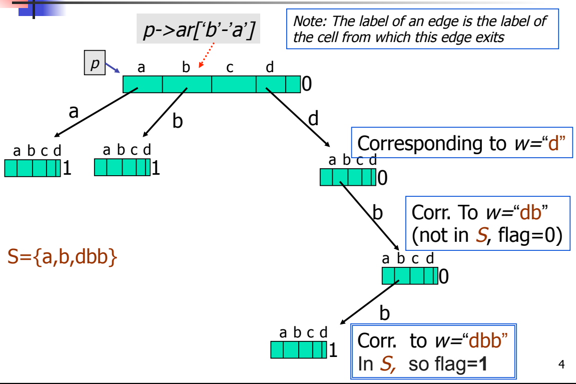
- A tree where each node is a struct consist
- Struct node {
  - char[4] *ar;
  - char flag ; /* 1 if a word ends at this node. Otherwise 0 */

  }



Rule:
Each node corresponds to a word ***w***
(***w*** which is in *S* iff the flag is 1)

A trie - example



Finding if word *w* is in the tree

p=root; *i* =0

While(1){

- If *w[i]* == '\0'   // we scanned all letters of *w*
  - then return the flag of *p* ; // True/False
- If the entry of p correspond to *w[i]* is NULL

  return false;
- Set *p* to be the node pointed by this entry, and set *i++;*

}

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Inserting a word *w*

Recall – we need to modify the tree so find(*w*) would return TRUE.

- Try to perform find(*w*).
  - If runs into a NULL pointers, create new node(s) along the path.
  - The *flag* fields of all new node(s) is 0.
- Set the flag of the last node to 1

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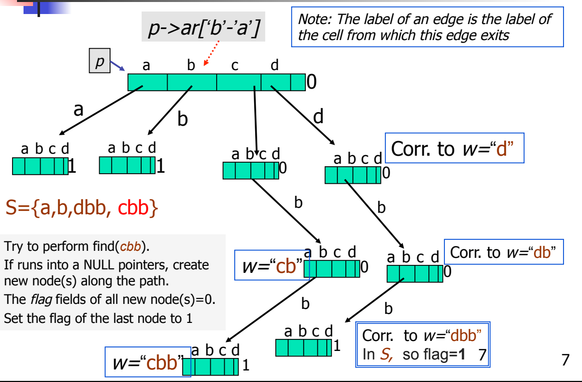
Inserting "cbb"



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Deleting a word *w*

- Find the node *p* corresponding to *w* (using `find' operation).
- Set the flag field of *p* to *0.*
- If *p* is dead (I.e. *flag==0* and all pointers are NULL) then free(*p*), set *p*=parent(*p*) and repeat this check.

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Space requirements

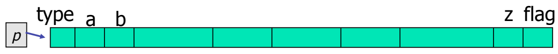
- Let ***m*** be is the sum of characters of all words in *S*
- The space required might be $\Theta(|\Sigma|\,m)$
- (for each letter of each words of *S*, we need an array of size $|\Sigma|$

(Might be an issue by itself, and might slow down performances)

Note – the letters are not stores explicit ally

Heuristics for space saving

- To save some space, if Σ is larger, there are a few heuristics we can use. Assume Σ={a,b..z} .
- We use two types of nodes
  - Type "A", which is used when the number of children of a node is more than 3



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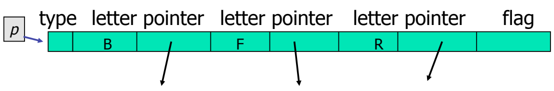
Heuristics for space saving

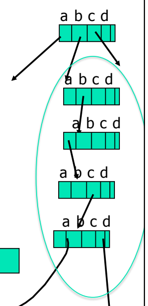
- Type "B" is used if there are 3 or less children:
- The "letter" of the child is also stored:

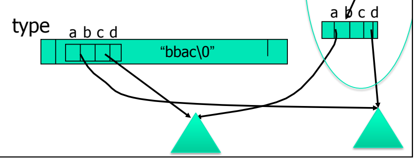


•The rule of the flag is the same as in type "A" nodes.
•We only store the 3 pointers, but we need to know to which letters they corresponds to

Another Heuristics – path compression

- Replace a long sequence of nodes that happens to have only a **single** child, with a single node (of type "pointer to string") that keeps a point to the next node, and a point to a string.





Suffix tree.

- Assume *B* (for book) is a long text.
- Want to preprocess *B*, so when a word *w* is given, we could quickly find if it is in *B*. (incremental search)
  - (as well as locations, how many etc)
- We can find it in O(|*w*|).
- Idea:
  - Consider *B* as a long string.
  - Create a trie *T* of all **suffixes** of *B*.
  - In addition to the flag (specifying if a word ends at node), we also stored the index in *B* where this word begins.
  - Example *B="aabab"*
    S={"aabab", "abab", "bab", "ab", "b"}

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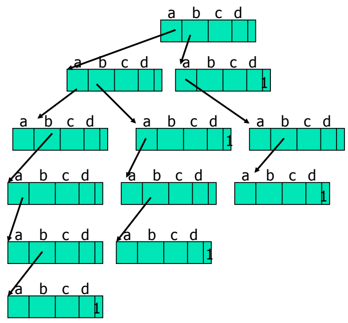
Suffix tree.

Example *B="aabab"* *S={"aabab", "abab", "bab", "ab", "b"}*



To know where a word appear in *B*, we store with each node the index of the beginning of the suffix in B.

(we can store only the first appearance of the word in the text)

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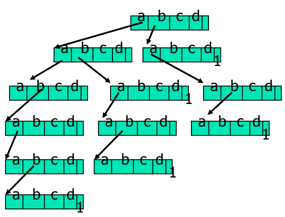
Size of suffix tree

Example *B="aabab"* *S={"aabab", "abab", "bab", "ab", "b"}*

Assume *n=|B|*.
Total length of all string $\Theta(n^2)$
Size of a node is $|\Sigma|$
So size of the tree is $\Theta(n^2 |\Sigma|)$.

Time to construct the tree $\Theta(n^2)$



Rather than a flag, we store the first index where the suffix appear

Example *B="aabab"*
S={"aabab", "abab", "bab", "ab", "b"}

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Size of suffix tree

12345
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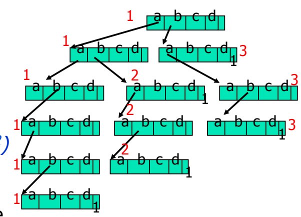

In addition to the flag, we store the first index (in the book) where the suffix starts (in red)

Example *B="aabab"*
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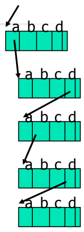
Suffix tries on a diet

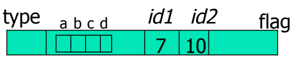


Def: a *shred is a* path from node *u to node v in the trie,* consisting of nodes of outdegree *1* (except maybe the last one) and *flag=0.*

Obs: There is a contiguous part of *B*, identical to the string the shred represents. We call this part the shred-string

We stores *B* itself as an array.

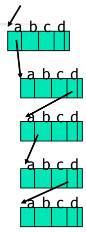
We use a new type of nodes, called *shred-nodes*, that maintain only the indexes of the first (*id1*) and last (*id2*) letters of the shred-string in *B*.



Example for shred of "adbd"

1 7 10
B="cadbdaadbd

Suffix tries on a diet - cont

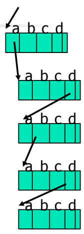


Algorithm for constructing a "thin" trie:

Given B – create an empty trie T, and insert all *n* suffixes of *B* into *T* --- generating a trie of size *$\Theta(n^2)$.*

Traverse the tries, and each time that a shred is seen, replace all nodes of the shred with a single shred-node.

Suffix tries on a diet - cont



Clearly the use of shred nodes saves some-but can we prove something ?

Observations: The # number of leaves of *T* is at most $\leq n$

(every leaf is the end of one prefix).

In addition there are nodes have a single child, but their *flag=1* (a suffixed have ended). We call them special nodes.

Observations: There are $\leq n$ special nodes.

- Thanks for patience.
- See you at the review

Suffix tries on a diet - cont

Lemma: Let *T* be a tree where each internal node has outdegree 2 or more, and *m* leaves. Then *T* has at most *m* internal nodes.

Back to thin suffix tries: *T* does not have **exactly** this property, but it is very close (no long shreds), so a "massaged" lemma still works, so

#internal_nodes is ≤ #leafs_nodes+#special_nodes,

But *#leafs_nodes + #special_nodes ≤*

#suffixes_of_B = n

So the size of the trie is only a constant more than the size of the book.

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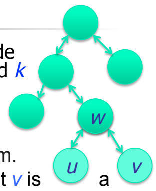
Proof of lemma (just FYI)



Lemma: Let *T* be a tree where each internal node has outdegree 2 or more, and *m* leaves and *k* internal nodes. Then $k \leq m$

Proof: Assume true for all trees with strictly less than *m* leaves, and assume *T has m* leaves.

Find a leaf *u* whose distance from root is maximum.

Assume it has has exactly one sibling *v.* Note that *v* is a leaf (why ?). Let *w* be their common parent.

Remove both *u* and *v* from *T.* Let *T'* be the resulting tree.

Let *k', m'* denote # internal nodes and leaves in *T'.* Now in *T'*

1. w is a leaf.
2. $m'=m-2+1=m-1.$
3. $k'=k-1.$
4. The outdegree of every internal node ≥2

From induction, $k' \leq m'$. *Hence* $k \leq m$

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