# CSc 445 — Homework #7
# Convex Hull, Closest Pair and Stable Matching
# Due 5/3/06

1. Prove that if $A$ and $B$ are convex sets, then $A \bigcap B$ is a convex sets.

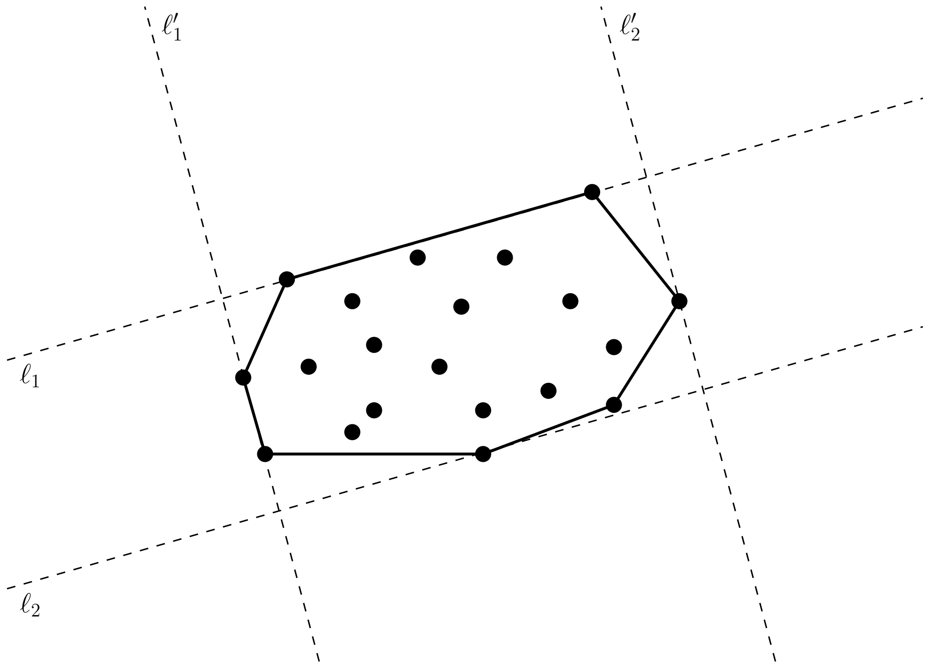



Figure 1: The lines $\ell_1$ and $\ell_2$ are the pair of parallel lines with minimum distance, whereas, $\ell_1'$ and $\ell_2'$ are not.

2. Let $S$ be a set of $n$ points. The *width* of $S$ is defined as the smallest distance between a pair of **parallel** lines $\ell_1, \ell_2$ such that $S$ lies above $\ell_1$ and below $\ell_2$. Show that if $\ell_1, \ell_2$ is the pair of the lines, then both $\ell_1, \ell_2$ pass through vertices of $CH(S)$ (the convex hull of $S$). Give an $O(n \log n)$ time algorithm for finding the width of $S$.

   Hint - Compute $P = CH(S)$. Rotate $P$ while maintaining a pair of *horizontal lines* $\ell_1, \ell_2$ such that $P$ touches both of them, and $P$ lies both above $\ell_1$ and below $\ell_2$. During the rotation process, update the distance between the lines accordingly. Show that there are $O(n)$ events to maintain.

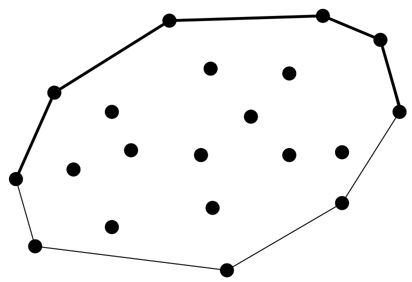


Figure 2: The upper convex hull of the convex hull is shown in bold.

3. Let $P$ be a convex polygon. The **upper convex hull** of $P$ are all the points which are on the boundary of the convex hull, and $P$ lies vertically below them

   (a) Let $S = \{p_1 \dots p_n\}$ be a set of $n$ points, and assume it is given to you sorted by the $x$-coordinate of the points (so $p_1$ is the leftmost point). Change the Graham-scan algorithm, so you can compute the upper convex hull in $O(n)$ time.

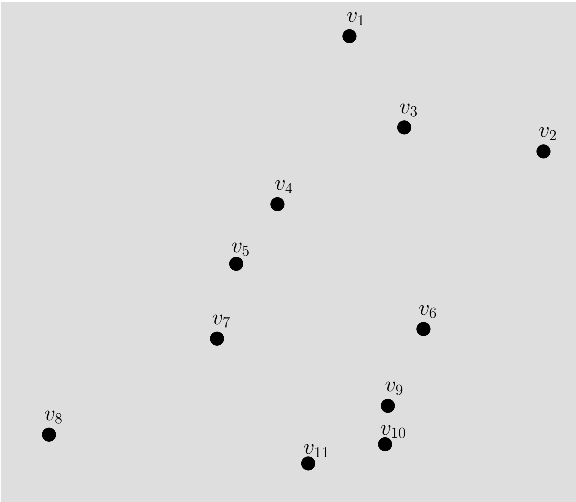
   (b) Assume that the coordinates of all points are integers between 1 and $n^2$, but now assume that they are given in an arbitrary order. Suggest an algorithm that enables you to compute $CH(S)$ in $O(n)$ time.

4. Explain how would you implement the stable marriage algorithm in your computer, so that the running time is $O(n^2)$ . Assume the input is the preferences list of each boy and each girl. Describe which data structures you are using, and which preprocessing would you do to the data before starting the actual algorithm.

5. Show an example of preferences list on which the stable marriage would run in $\Theta(n^2)$ and prove the bound holds.

   Show an example of preferences list on which the stable marriage would run in $O(n)$ and prove the bound holds.

6. Run Graham scan algorithm on the points on the figure on the next page. Twelve copies of the figure are provided so that for each step you can draw the line segments of convex hull so far and the state what happens to the contents of stack for that step.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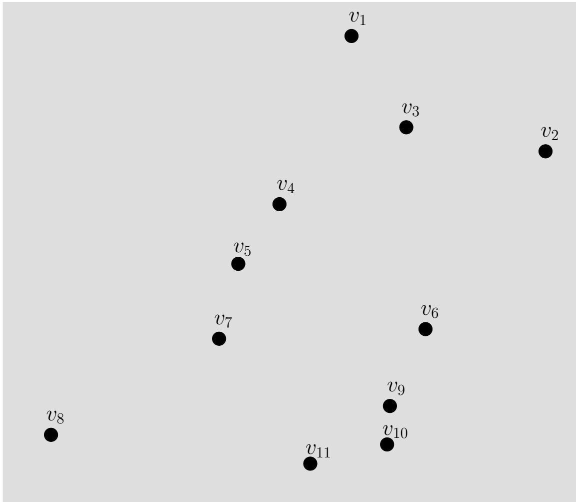
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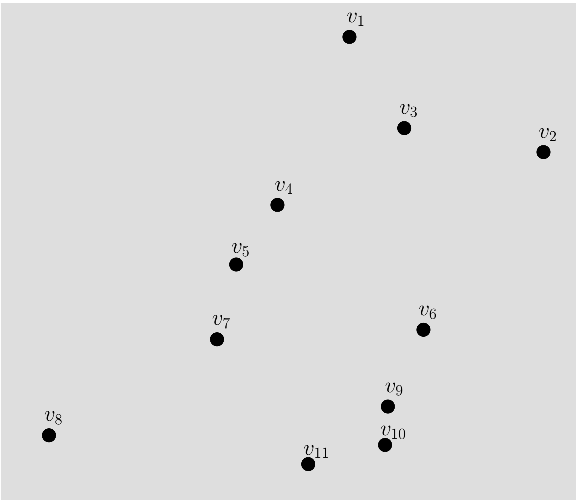
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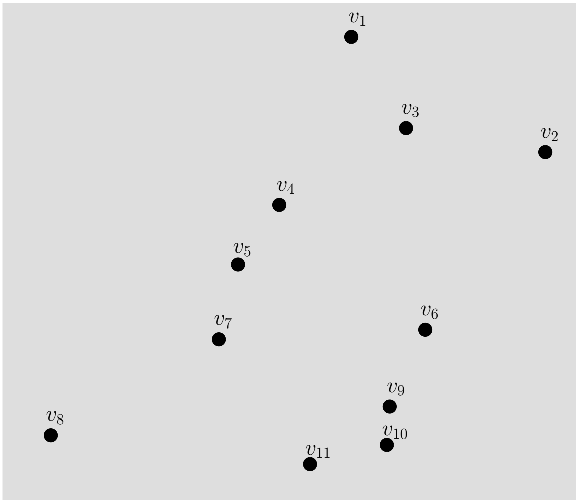
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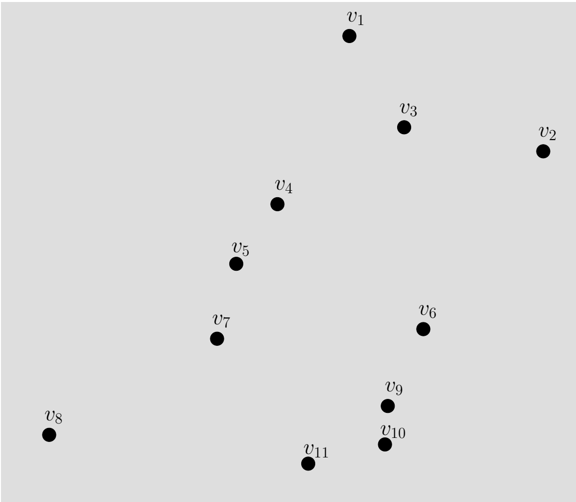
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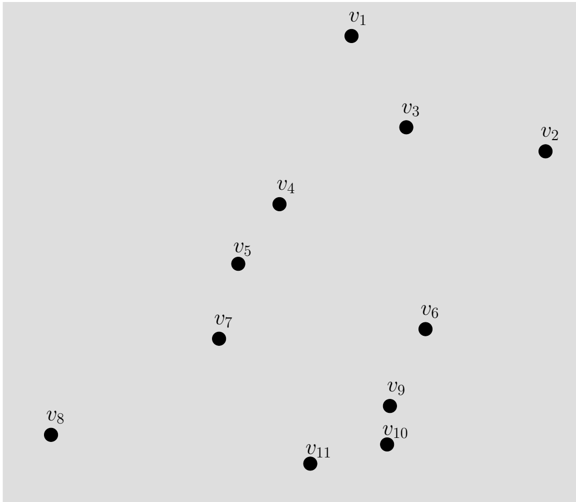
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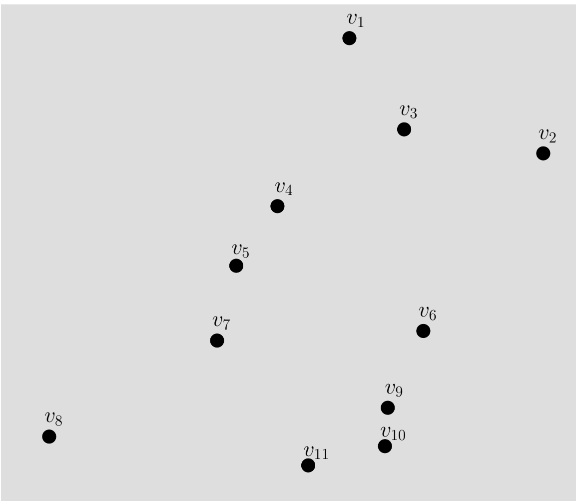
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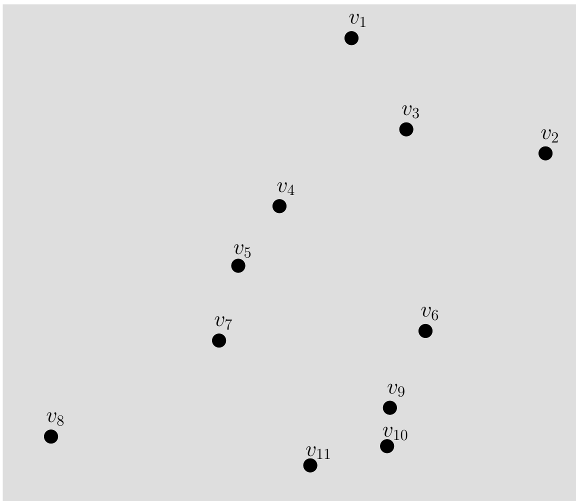
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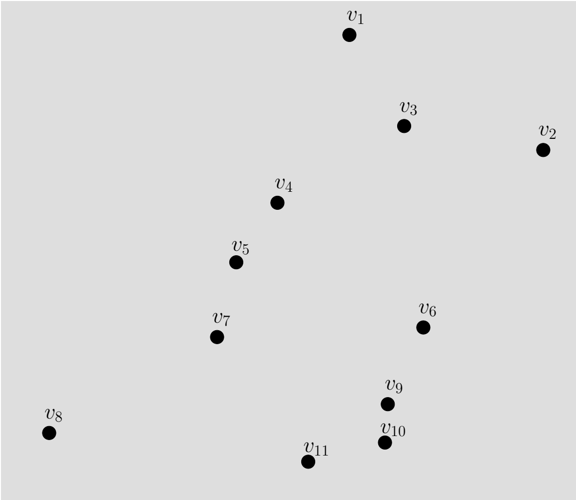
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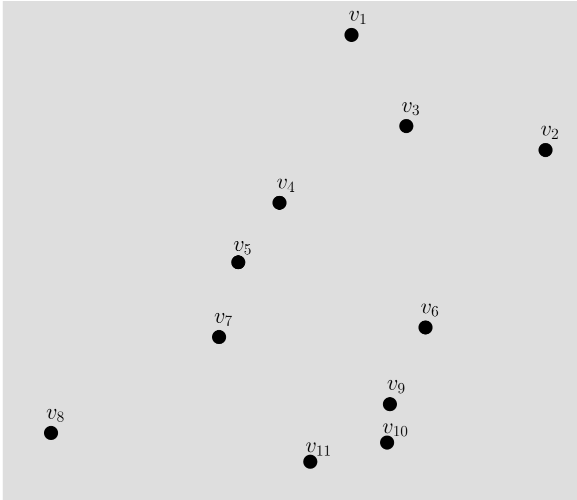
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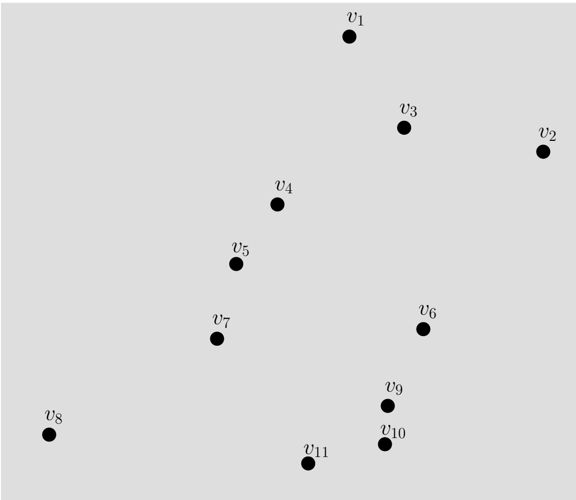
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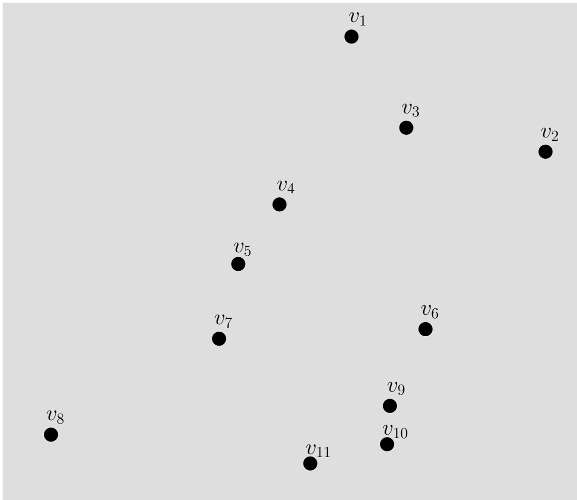
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