Cs445 — Homework #7 More flow, approximation and computational geometry.

Due: 2 May 2007 during class meeting.

Instructions. All assignments are to be completed on separate paper. Use only one side of the paper. Assignments will be due at the beginning of class. To receive full credit, you must show all of your work. All questions have the same value.

1. Let $G(A \cup B, E)$ be a bipartite graph, and let G' be the network flow problem produced from G in the way described in the slides. Let n be a given integer. Show an example of G, where $n = |A| = |B|$, such that if we use the Edmonds-Karp algorithm to find max-flow on this problem, then it might be that all the augmenting paths that the algorithm finds are of length 3 (in the number of edges), excluding the last augmenting path that the algorithm uses whose length is at least $2n - 2$.

2. Suggest an algorithm that accepts a network flow problem (that is, the graph together with the capacities of the edges), and finds a cut with minimum capacity in the network.

3. Let $G(V, E)$ be a given graph. Explain how to find a factor 2 approximation to the problem of finding a shortest tour (that is, a cycle, not necessarily simple, that visits every vertex **at least** once). Do you need the triangle inequality for this problem?

   Reminder — the definition of a factor k approximation, or just a k-approximation, is found on pages 1022-1023 of CLRS.

4. Let $C = \{A_1, \ldots, A_m\}$ be a collection of subsets of integers, where each A_i contains exactly 3 different numbers from the range $1, \ldots, n$. So for example, $C = \{(2, 4, 6), (1, 2, 7), (6, 14, 19)\}$. Suggest an algorithm that finds a factor 3 approximation to the following problem. Find a set X of integers, whose cardinality is as small as possible, and every member of C contains at least one number in X.

The running time of algorithm should be $O(m+n)$. You might want to find first an $O(m^2+n)$, and then explain how to improve the running time using the appropriate data structure.

Reminder — a factor 3 approximation means that $|X| \leq 3|X^*|$, where X^* is the smallest-cardinality set of integers with the property that every A_i has a non-empty intersection with X^*.

5. You are given two sets A and B of points in the plane, where $n = |A| = |B|$. You are also given a real value r. Describe an $O(n^3)$ time algorithm that determines if there exists a one-to-one matching between the points of A to the points of B, and every point $a_i \in A$ is matched to a point $b_j \in B$ whose distance from a_i is at most r.

6. Draw your name or birthday so every digit and character is drawn as a set of line segments, and then run the line sweep algorithm to find if there is a pair of segments that intersect. Feel free to express your artistic tendencies in the way you draw.

7. You are given a set S of segments, each of which is either horizontal or vertical. Describe an $O(n \log n)$ time algorithm to find if two lines intersect. Note that we assumed in the description of the line-sweep algorithm that no segment is vertical.

8. Let $P = (\ (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)\ )$ and $P' = (\ (x'_1, y'_1), (x'_2, y'_2) \ldots (x'_n, y'_n)\ )$ be two polygonal paths, given by their vertices, (so each (x_i, y_i) is a vertex of P, and the ith segment of P is defined by $\overline{(x_i, y_i), (x_{i+1}, y_{i+1})}$). Assume that $x_1 \leq x_2 \leq \ldots \leq x_n$ and $x'_1 \leq x'_2 \leq \ldots \leq x'_n$. Explain how to find in $O(n)$ time if there is any pair of segments, one of P and one of P', that intersect each other.

9. Give an example of a set S of n segments, where no two segments intersect, but when we apply the line sweep algorithm on this problem, one of the segments $e \in S$ is checked for intersection against every other segment in S. Prove your answer.

10. Is it possible to generate an algorithm that accepts a set S of n segments and another segment e, and in time $O(n)$ computes all intersections of segments from S with e, and reports them in the order they appear along e? (Note that we don't care about intersections of one segment of S with another segment of S).