CSc 445: Homework Assignment 3

Assigned: Monday Feb 18 2008,
Due: 10:30 AM, Monday March 3 2008

Clear, neat and concise solutions are required in order to receive full credit. Revise your work carefully before submission, and consider how your work is presented. If you cannot solve a particular problem, state this clearly in your write-up, and write down only what you know to be correct. For involved proofs, first outline the argument and then delve into the details.

1. (20 pts) Traditionally, we talk about numbers in decimal, binary, octal, and other (positive) bases. However, it is possible to use negative numbers, and there are several advantages in this approach. For example, there is no need for a sign bit, as both negative and positive numbers can be represented. Consider the case where -2 is used as a base. The digits needed are 0 and 1 as in regular binary notation but the interpretation is different; 1101 is -3 and 10010 is 14 because:

$$(a_n \dots a_2 a_1 a_0) = a_n(-2)^n + \cdots + a_2(-2)^2 + a_1(-2)^1 + a_0.$$

   (a) What is the procedure for finding the base -2 notation of a given (decimal) number?

   (b) Prove that the 2^n possible bit patterns in an n-bit word in base -2 uniquely represent all integers in a certain range. (Hint: first determine the range and then use induction).

   (c) Determine the rules for addition and subtraction in base -2.

   (d) Determine the rules for negating a number in base -2.

   (e) Determine the rules for multiplication in base -2.

   (f) Determine the rules for converting between binary and base -2 numbers.

2. (10 pts) Consider the following notation for representing integers with two digits 0 and 1: 0 and 1 are represented by 0 and 1; to get the representations for 0,1,2,3, we take the above list, and append a copy of it in reverse to get 0, 1, 1, 0 and prepend 0's to each element in the original list and 1 to each element in the reversed list to get (00, 01, 11, 10). To get the representations for $0, 1, \dots, 7$ we repeat the same process and obtain $(000, 001, 011, 010, 110, 111, 101, 100)$. Clearly, this process generalizes and we can represent any integer with a unique sequence of 0's and 1's in this notation. Note that, unlike binary or base -2 notation, this encoding has the property that the representation of any two consecutive integers differs in exactly one bit.

   (a) Describe the rules to convert between this notation and regular binary notation, only using the XOR operation and shifting.

   (b) Describe the rules for incrementing a counter in this notation.

   (c) Consider the numbers $0, 1, \dots, 2^k - 1$ in this representation as vertices of a graph and connect with an edge every pair that differs in exactly one bit, for $k = 2, 3, 4$. In each of these cases, how many ways are there to traverse the graph so that each vertex is visited exactly once? (Note that traversals that are the same as an already listed traversal, up to a cyclic rotation are not counted.)

3. (10 pts) This is a modification of the GOOG problem from the last homework assignment. This time we would like to study the connections between "good press coverage" and the performance of the stock. In this model, we store the **difference from yesterday's price per share** over n days. The goal of this modeling exercise is to find the best **time range** (which is likely to be tied to some very good news-story about the company) determined by the largest sum of the price-differences over all possible contiguous time ranges.

(a) Design and analyze a $O(n^3)$ algorithm for the problem and argue correctness and running time.

(b) Design and analyze a $O(n \log n)$ divide and conquer algorithm for the problem and argue correctness and running time.

4. (10 pts) As you know, recursive programs can be inefficient (recall the Fibonacci program from the first homework assignment). As described in class, RandomizedSelect finds the i-th order statistic in expected $O(n)$ time using a recursive algorithm. Modify the algorithm so it does not use recursion. Does the algorithm still run in expected $O(n)$ time?

5. (10 pts) Prove that $\lceil 3n/2 \rceil - 2$ is the lower bound for the number of comparisons needed to find both the min and max in an array of n distinct integers.

6. (10 pts) Recall the deterministic $O(n)$ time Select algorithm from class. Recall that it relies on dividing the input array into groups of 5 elements. As we discussed in class, the constant 5 plays an important part in the analysis. Consider the same algorithm, where we divide in groups of 3 instead. Does the algorithm still run in linear time? How about if we use groups of size 7? For both cases carefully prove your claims.

7. (10 pts) Given two arrays each containing n sorted elements, design and analyze a $O(\lg n)$-time algorithm to find the median of all $2n$ elements.

8. (10 pts) Haliburton wants to minimize the cost of rebuilding the roads in Iraq. They've decided that the best strategy is to build one straight road connecting Syria (in the north-west) to Kuwait (in the south-east) and then connect all other cities to the main road by perpendicular straight roads. Assuming that the orientation of the main road is at 45° and knowing the precise coordinates of the n major cities in Iraq that will be connected, design and analyze an efficient algorithm for minimizing the total road-miles that need to be constructed.

9. (10 pts) Given an array of n numbers, we would like to find an ordered subset of the elements of the array which are in increasing order. Among all such subsets we are interested in the largest. Design and analyze an $O(n^2)$ divide and conquer algorithms that finds the size of the largest such subset.

Extra Credit: What is the next number in the sequence 1, 11, 21 and why?