CSc 445: Homework Assignment 5

Assigned: Wednesday March 31 2008,
Due: 10:30 AM, Monday April 14 2008

Clear, neat and concise solutions are required in order to receive full credit. Revise your work carefully before submission, and consider how your work is presented. If you cannot solve a particular problem, state this clearly in your write-up, and write down only what you know to be correct. For involved proofs, first outline the argument and then delve into the details.

For all dynamic programming problems follow the 4-step procedure described in class (and don't forget to write the pseudocode for your algorithm and analyze its running time):

1. **Optimal substructure;**
2. **Recursive formula that characterizes the optimal solution in terms of optimal solutions to smaller problems;**
3. **Computing the optimal costs (describing the table that you'll use and how you're going to fill it up (e.g., row major order, diagonal order, etc.);**
4. **Reconstructing the optimal solution from the table.**

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1. (20 pts) Recall the Matrix Chain Multiplication problem.
   (a) Prove that any parenthesization of n elements has exactly $n-1$ pairs of parenthesis.
   (b) Define $R(i,j)$ to be the number of times that table entry $m[i,j]$ is referenced which computing other entries in a call of MatrixChainOrder (the bottom-up dynamic programming version we discussed in class). Show that
   $$\sum_{i=1}^{n}\sum_{j=i}^{n} R(i,j) = \frac{n^3-n}{3}$$
   (Hint: take a look at equation A.3 in the textbook).

2. (10 pts)

   Recall the Appalachian trail problem. After a fund-raising campaign, we have money to improve some of the shelters along the trail. However, not all shelters are equally in need of repair. A knowledgeable hiker has assigned them integer values $v_i > 0$ (the higher the number, the greater the need of repair). Since experienced hikers can easily cover 10 miles per day, the Appalachian Trail Committee has decided that no two shelters at a distance of less than 10 miles should be improved together. Design and analyze an efficient dynamic programming algorithm that finds the best set of shelters.

3. (20 pts)
   (a) Design and analyze an efficient dynamic programming algorithm for the optimal polygon triangulation problem, where we use the weight function: $w(\Delta v_i v_j v_k) = |v_i v_j| + |v_j v_k| + |v_k v_i|$, where $|v_i v_j|$ is the Euclidean distance from v_i to v_j.
   (b) Design a memoized version of the algorithm, that is, an efficient top-down recursive algorithm that uses a look-up table. (Hint: you may want to start with a simple recursive algorithm that may not have a good running time.)

4. (10 pts) Recall the GOOG problem from HW3 ("good press coverage" and the performance of the stock). In this model, we store the **difference from yesterday's price per share** over n days. The goal of this modeling exercise is to find the best **time range** (which is likely to be tied to some very good news-story about the company) determined by the largest sum of the price-differences over all possible contiguous time ranges. Design and analyze an efficient dynamic programming algorithm for the problem.

5. CLRS 15-1 (20 pts)

6. CLRS 15-2 (20 pts)

Extra Credit: I climb stairs one or two steps at a time. If there are n steps to be climbed, how many different ways are there to climb them?