CSc 445: Homework Assignment 6

Assigned: Monday April 14 2008,
Due: 10:30 AM, Monday April 28 2008

Clear, neat and concise solutions are required in order to receive full credit. Revise your work carefully before submission, and consider how your work is presented. If you cannot solve a particular problem, state this clearly in your write-up, and write down only what you know to be correct. For involved proofs, first outline the argument and then delve into the details.

1. (10 pts) Given a directed graph $G = (V, E)$, $|V| = n$, and its adjacency matrix representation, design a $O(n)$ time algorithm for determining whether there exists a vertex of G with in-degree=$n-1$ and out-degree=0.

2. (10 pts) We are given n college basketball teams and a schedule of games with k matches between these teams. You'd like to root for at least one team in each game and so you decide to divide the teams into two categories: those you root for (e.g. Arizona) and teams you root against (e.g. Duke). Design and analyze a $O(n+k)$ time algorithm to determine whether it is possible to divide the teams into two such groups. If it is possible, your algorithm should also find the assignment.

3. (20 pts) Given an undirected graph $G = (V, E)$, we would like to "physically" explore it. That is, unlike in the traditional implementations of BFS and DFS, we cannot magically "teleport" from one node in the graph to another. In the myth of Theseus and Ariadne, the graph in question was a labyrinth which housed a monster, the Minotaur.

   (a) Now suppose that we have Ariadne's thread (of finite length!) and we decide to tie it to a tree near the entrance of the labyrinth. Show that there exist labyrinths (graphs) that would force us to traverse $|V|^2$ edges, if we are using BFS, even if the set of edges is $|E| = O(|V|)$.

   (b) Using the same setting (thread tied to the entrance) show that there exist labyrinths (graphs) that would make us fail to explore the entire graph if we are using DFS.

   (c) Instead of Ariadne's thread we decide to take some bread (ala Henzel and Gretel) and use crumbs to mark our passage. Assuming that the number of crumbs is proportional to the number of vertices and edges of the graph, design an algorithm that traverses each edge in E once in each direction and runs in $O(|V| + |E|)$ time.

4. (10 pts) Design and analyze a $O(|V| + |E|)$-time algorithm for topological sorting of directed acyclic graphs that uses the idea of repeatedly finding a vertex of in-degree 0 and removing it together with its outgoing edges.

5. (10 pts) An independent set of of an undirected graph $G = (V, E)$ is a subset I of V such that no two vertices in I are adjacent. A maximal independent set M is an independent set such that, if we were to add any additional vertex to M, then it would not be independent. Give an efficient algorithm that computes a maximal independent set for a graph G. Is computing the maximal independent set an approximation algorithm for the maximum independent set problem? If so, what is the approximation ratio?

6. (10 pts) Your big, multinational, and high-paying company decides to update its computer network. The network is set up as a tree and the idea is to purchase one expensive file server. The problem you have been assigned is to determine the location of the file server. Since the transmission time on a link is dominated by the link setup and synchronization, the cost of a data transfer is proportional to the

number of links used. Hence it is desirable to choose a "central" location for the file server. That is, given a tree $T = (V, E)$ you need to find the node v in the tree that minimizes the length of the longest path from v to any other node. Design an efficient algorithm that finds the center of the tree. Is the center unique? If not, how many distinct centers can there be? Prove your claims.

7. (20 points) Recall that an undirected graph is biconnected if for any pair of nodes there exist at least two paths between them. Design and analyze an efficient algorithm for computing the biconnected components of given graph.

8. (10 points) CLRS 23-4 (Alternative MST algorithms)

Extra Credit: I have three boxes in my office. One of them contains a piece of paper with the letter "A" on it and the other two have the letter "F". I offer the gamblers in the class a chance to get an automatic "A" on the final if they select the box with the "A". Joe, a student in the class, shows up in my office and picks a box. Before opening the chosen box, I open one of the remaining boxes, revealing an "F". I offer Joe the choice to change his mind. Should he?