Tries and suffixes trees

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Trie: A data-structure for a set of words

All words over the alphabet $\Sigma = \{a, b, \ldots, z\}$. In the slides, let say that the alphabet is only $\{a, b, c, d\}$

$S$ – set of words $= \{a, aba, a, aca, addd\}$

Need to support the operations

- $\text{insert}(w)$ – add a new word $w$ into $S$.
- $\text{delete}(w)$ – delete the word $w$ from $S$.
- $\text{find}(w)$ is $w$ in $S$?

Future operation:

- Given text (many words) where is $w$ in the text.

The time for each operation should be $O(k)$, where $k$ is the number of letters in $w$

Usually each word is associated with addition info – not discussed here.

Trie (Tree+Retrieve) for $S$

A tree where each node is a struct consist

Struct node {
  char[n] *ar;
  char flag; /* 1 if a word ends at this node.
  Otherwise 0 */
}

Rule: Each node corresponds to a word $w$ (which is in $S$ if the flag is 1)
A trie - example

Note: The label of an edge is the label of the cell from which this edge exits.

Corr. To \( w = \text{"dbb"} \)

S = \{a, b, d, bb\}

Finding if word \( w \) is in the tree

\[ p = \text{root}; \ i = 0 \]

\( \text{While(1)}(\)

\( \quad \text{If } w[i] = \text{"\0"} \quad // \text{we scanned all letters of } w \)

\( \quad \text{then return the flag of } p; // \text{True/False} \)

\( \quad \text{If the entry of } p \text{ correspond to } w[i] \text{ is NULL} \)

\( \quad \text{return false;} \)

\( \quad \text{Set } p \text{ to be the node pointed by this entry, and set } i++; \)

\( \}\)

Inserting a word \( w \)

Recall – we need to modify the tree so \( \text{find}(w) \) would return TRUE.

\( \bullet \) Try to perform \( \text{find}(w) \).

\( \quad \bullet \) If runs into a NULL pointers, create new node(s) along the path.

\( \quad \bullet \) The flag fields of all new node(s) is 0.

\( \bullet \) Set the flag of the last node to 1
Inserting “cbb”

$S = \{a, b, dbb, cbb\}$

Corr. to $w = "dbb"

Note: The label of an edge is the label of the cell from which this edge exits.

Try to perform `find(cbb)`. If runs into a NULL pointers, create new node(s) along the path. Set the flag of the last node to 1.

Deleting a word $w$

1. Find the node $p$ corresponding to $w$ (using `find` operation).
2. Set the flag field of $p$ to 0.
3. If $p$ is dead (i.e. $flag==0$ and all pointers are NULL), then free($p$), set $p=parent(p)$ and repeat this check.

Space requirements

- Let $m$ be is the sum of characters of all words in $S$
- The space required might be $|S| \cdot m$
- (for each letter of each words of $S$, we need an array of size $|X|$)

(Might be an issue by itself, and might slow down performances)

Note – the letters are not stores explicitly.
Heuristics for space saving

To save some space, if $\Sigma$ is larger, there are a few heuristics we can use. Assume $\Sigma=\{a, b, .. z\}$.

We use two types of nodes:

- Type "A", which is used when the number of children of a node is more than 3.

<table>
<thead>
<tr>
<th>type</th>
<th>a</th>
<th>b</th>
<th>z</th>
<th>flag</th>
</tr>
</thead>
</table>

Note – the letters are not stored explicitly.

Heuristics for space saving

- Type "B" is used if there are 3 or less children:
- The "letter" of the child is also stored:

<table>
<thead>
<tr>
<th>type</th>
<th>letter pointer</th>
<th>letter pointer</th>
<th>letter pointer</th>
<th>flag</th>
</tr>
</thead>
</table>

- The rule of the flag is the same as in type "A" nodes.
- We only store the 3 pointers, but we need to know to which letters they correspond to.

Another Heuristics – path compression

- Replace a long sequence of nodes that happens to have only a single child, with a single node (of type “pointer to string”) that keeps a point to the next node, and a point to a string.

<table>
<thead>
<tr>
<th>type</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>flag</th>
</tr>
</thead>
<tbody>
<tr>
<td>type</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>flag</td>
</tr>
</tbody>
</table>
Suffix tree.

Assume $B$ (for book) is a long text. We can find it in $O(|w|)$. Idea:
- Consider $B$ as a long string.
- Create a trie $T$ of all suffixes of $B$. In addition to the flag (specifying if a word ends at node), we also stored the index in $B$ where this word begins.

Example $B$="aabab"
$S=${"aabab", "abab", "bab", "ab", "b"}

Size of suffix tree

Assume $n=|B|$. Total length of all string $\Theta(n^2)$ Size of a node is $|\Sigma|$ So size of the tree is $\Theta(n^2 |\Sigma|)$.

To know where a word appear in $B$, we store with each node the index of the beginning of the suffix in $B$. (we can store only the first appearance of the word in the text.)

Rather than a flag, we store the first index where the suffix appear
**Size of suffix tree**

Example $B = \text{"aabab"}$  $S = \{\text{"aabab"}, \text{"abab"}, \text{"bab"}, \text{"ab"}, \text{"b"}\}$

Assume $n = |B|$.  Total length of all string $\Theta(n^2)$

Size of a node is $|\Sigma|$

So size of the tree is $\Theta(n^2 |\Sigma|)$.

Time to construct the tree $\Theta(n^2)$

In addition to the flag, we store the first index (in the book) where the suffix starts (in red)

Example $B = \text{"aabab"}$  $S = \{\text{"aabab"}, \text{"abab"}, \text{"bab"}, \text{"ab"}, \text{"b"}\}$

**Suffix tries on a diet**

Def: a shred is a path from node $u$ to node $v$ in the trie, consisting of nodes of outdegree 1 (except maybe the last one) and flag=0.

Obs: There is a contiguous part of $B$, identical to the string the shred represents. We call this part the shred-string.

We store $B$ itself as an array.

We use a new type of nodes, called shred-nodes, that maintain only the indexes of the first (id1) and last (id2) letters of the shred-string in $B$.

<table>
<thead>
<tr>
<th>type</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>id1</th>
<th>id2</th>
<th>flag</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Example for shred of "adbd"

$B = \text{"cadbdaadbd"}$

**Suffix tries on a diet - cont**

Algorithm for constructing a "thin" trie:

Given $B$ – create an empty trie $T$, and insert all $n$ suffixes of $B$ into $T$ --- generating a trie of size $\Theta(n^2)$.

Traverse the tries, and each time that a shred is seen, replace all nodes of the shred with a single shred-node.
Clearly the use of shred nodes saves some—but can we prove something?

**Observations:** The number of leaves of $T$ is at most $\leq n$

(every leaf is the end of one prefix).

In addition there are nodes have a single child, but their $\text{flag}=1$ (a suffixed have ended). We call them special nodes.

**Observations:** There are $\leq n$ special nodes.

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Thanks for patience.

See you at the review

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**Lemma:** Let $T$ be a tree where each internal node has outdegree 2 or more, and $m$ leaves. Then $T$ has at most $m$ internal nodes.

Back to thin suffix tries: $T$ does not have exactly this property, but it is very close (no long shreds), so a "massaged" lemma still works, so

\[ \#\text{internal\_nodes} \leq \#\text{leafs\_nodes} + \#\text{special\_nodes}, \]

But

\[ \#\text{leafs\_nodes} + \#\text{special\_nodes} \leq \]

\[ \#\text{suffixes\_of\_B} = n \]

So the size of the trie is only a constant more than the size of the book.
**Proof of lemma (just FYI)**

**Lemma:** Let \( T \) be a tree where each internal node has outdegree 2 or more, and \( m \) leaves and \( k \) internal nodes. Then \( k \leq m \).

**Proof:** Assume true for all trees with strictly less than \( m \) leaves, and assume \( T \) has \( m \) leaves.

Find a leaf \( u \) whose distance from root is maximum. Assume it has exactly one sibling \( v \). Note that \( v \) is a leaf (why?). Let \( w \) be their common parent. Remove both \( u \) and \( v \) from \( T \). Let \( T' \) be the resulting tree.

Let \( k', m' \) denote \# internal nodes and leaves in \( T' \). Now in \( T' \):
- \( w \) is a leaf.
- \( m' = m - 2 + 1 = m - 1 \).
- \( k' = k - 1 \).
- The outdegree of every internal node \( \geq 2 \).

From induction, \( k' \leq m' \). Hence \( k \leq m \).