This homework is due Monday October 2 by 4:00pm. Please place it in the box marked "CSc 473" in the department mail room (Gould-Simpson 713). Questions are drawn from the material in Sections 1.3 and 1.4 of the text on regular expressions and nonregular languages.

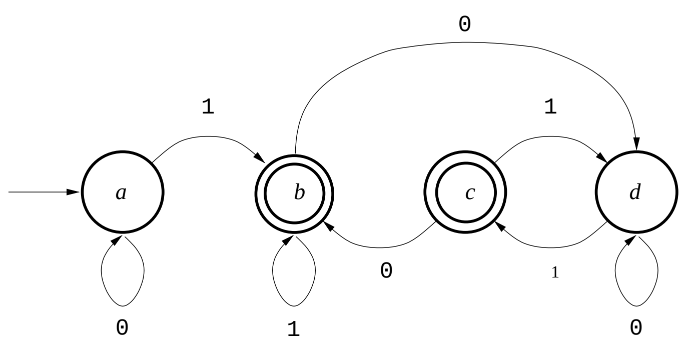
This homework is worth 100 points. In questions with several parts that do not specify the points for each part, each part has equal weight.

Remember to write on just one side of a page, do not use scrap paper, put your answers in the correct order, and staple your pages together. If you can't solve a problem, state this, and write only what you know to be correct. Neatness and conciseness count.

(1) **(Converting regular expressions to automata)** (5 points) Convert the following regular expression to an equivalent NFA. Proceed bottom-up over the expression tree, and do not simplify the resulting NFA.

$$\Big((\varepsilon+\mathtt{0})\,\mathtt{1}^* \;+\; (\mathtt{0}+\mathtt{1})^*\,\mathtt{1}\Big)^*$$

(2) **(Converting automata to regular expressions)** (5 points) Convert the following DFA over alphabet $\{\mathtt{0},\mathtt{1}\}$ to an equivalent regular expression. Show your intermediate GNFAs, do not simplify the final regular expression, and eliminate states in the order d, c, b, a.



(3) **(Deriving regular expressions)** (20 points) Give regular expressions that describe each of the following languages, which are over the alphabet $\{\mathtt{0},\mathtt{1}\}$. Explain how you constructed your regular expression for each language.

(a) $\Big\{w \;:\; w \text{ contains substrings } \mathtt{010} \text{ and } \mathtt{101}\Big\}$

(b) $\Big\{w \;:\; w \text{ does not contain substring } \mathtt{0110}\Big\}$

(c) $\Big\{w \;:\; w \text{ has an even number of } \mathtt{0}\text{'s and an even number of } \mathtt{1}\text{'s}\Big\}$

(d) $\Big\{w \;:\; w \text{ has the same number of occurrences of } \mathtt{10} \text{ and } \mathtt{01}\Big\}$

Note that in Parts (a) and (d), occurrences of substrings can overlap.

(4) **(Determining regularity)** (70 points) Prove or disprove whether each of the following languages is regular. All languages are over the alphabet $\{\texttt{0}, \texttt{1}\}$. Below, w^n where w is a string and n is a nonnegative integer, denotes the concatenation of n copies of w. Also, w^R denotes the reverse of string w.

(a) $\left\{\texttt{0}^{\lceil (3n+1)/2 \rceil} \;:\; n \geq 0\right\}$

(b) $\left\{\texttt{0}^{(3n+1)^2} \;:\; n \geq 0\right\}$

(c) $\left\{(\texttt{10})^n\,(\texttt{01})^n\,\texttt{0} \;:\; n \geq 0\right\}$

(d) $\left\{(\texttt{10})^n\,\texttt{1}\,(\texttt{01})^n\,\texttt{0} \;:\; n \geq 0\right\}$

(e) $\left\{w^R\,w \;:\; w \in L\right\}$, where L is any regular language

(f) **(bonus)** $\left\{w \;:\; w^R\,w \in L\right\}$, where L is any regular language

(g) $\left\{x\,y \;:\; \exists x \in A \text{ and } \exists y \in B \text{ such that } x \text{ and } y \text{ have the same number of } \texttt{0}\text{'s}\right\}$, where A and B are any regular languages

(h) $\left\{x \;:\; \exists x \in A \text{ and } \exists y \in B \text{ such that } x \text{ and } y \text{ have the same number of } \texttt{0}\text{'s}\right\}$, where A and B are any regular languages

In Parts (e) through (h), you need to prove or disprove whether the given language is regular, *for all* regular languages L, A, and B. To *disprove* the language is regular, make a specific choice for the regular languages L, A, and B, and use the **pumping lemma** to prove that the resulting language is not regular. To *prove* the language is regular, give a **construction** of a finite automaton that recognizes the given language, assuming you have finite automata that recognize languages L, A, and B.

(Hints: In Parts (f) and (h), it may be useful to know the *product construction* described in the proof on pages 45–46 of the text. This construction can be used to show that the *intersection* of any two regular languages is regular, as described in the footnote on page 46. In Part (f), it may be useful to show that the *reverse* of a regular language is regular.)

Note that Problem (4)(f) is a bonus question. It is not required (except for the honors section) and its points are not added to regular points.