NAME____________________________**Score/80**______**Grade**______

CSc/Math 473 **EXAMPLE final exam**

Do eight (8) problems. All problems have equal weight. Write all answers on this examination, and submit it in the envelope at the end of the exam.

TIME = 2 hours.

You can do ONE additional problem for extra credit. If you do so, *circle the extra credit problem* you attempted in the table below.

NAME											
Problem	1	2	3	4	5	6	7	8	9	10	Σ
Maximum	10	10	10	10	10	10	10	10	10	10	80+10
Score											

1. Irregularity

Let L be the following language over $\Sigma = \{a, b, c\}$:

$$L = \{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = |v| \}$$

Show that L is not a regular language.

2. True or False

Be very careful to read what is actually asserted.

(a)______The Turing-recognizable languages are closed under complementation.

(b)______The Turing-decidable languages are closed under complementation.

(c)______The following problem is unsolvable: given context-free grammar G, determine whether or not $L(G) = \emptyset$.

(d)______Any language in Σ^* with a context-free complement is decidable.

(e)______The set $\{ <M> \mid M \text{ is a TM} \}$ is decidable.

(f)______The set $\{ <M> \mid <M> \in L(M) \}$ is decidable.

(g)______It is decidable, given a TM M and configurations $C_1 = uqav$ and $C_2 = ua'q'v$, whether $C_1 \vdash_M C_2$.

(h)______It is decidable, given a TM M and configurations $C_1 = uqav$ and $C_2 = q'\varepsilon$, whether $C_1 \vdash_M^* C_2$.

(i)______It is decidable, given a CF grammar G and a string w, whether $w \in L(G)$.

(j)______It is decidable, given a right-linear grammar G and a string w, whether $w \in L(G)$.

3. Hierarchy

In each part below, describe (using set notation) a specific language that has the given property. A response like "A_{TM}" is not sufficient: you must define the set of strings precisely.

(a) not Turing-recognizable but with a Turing-recognizable complement

(b) Turing-recognizable but not Turing-decidable

(c) Context-free but not Regular

(d) Turing-recognizable but not Context-free

(e) Turing-recognizable with a non-Turing-recognizable complement

4. Relational Calculus

Let $R \subseteq A \times A$. R is a *quasi-order* if it is both irreflexive ($R \subseteq \overline{I_A}$) and transitive ($R \circ R \subseteq R$).
Prove: R is a quasi-order if and only if $R \cap R^{-1} = \varnothing$ and $R = R^{+}$.

5. Determination

Construct a DFA equivalent to the NFA given below, using the Rabin-Scott algorithm. Show any intermediate work. Label all the DFA states appropriately. Here ε denotes the empty string.

6. Unsolvable Problem

A TM is said to be *tidy* if, whenever it halts for some input, it halts with its tape completely blank.

Show that the following problem is unsolvable:

Given: A Turing Machine M

Question: Is M a tidy TM?

7. Incrementing by TM

Design a TM with tape alphabet $\Sigma = \{0, 1, \#\}$ that, given a binary representation of a number, computes the binary representation of its successor. For example, if M is started in configuration

$\underline{\# s}110101111\#$

then it halts in configuration

$\#110110000h\underline{\#}$

(Here for emphasis the character scanned by the read/write head is designated by an underscore.)

You may give either a detailed TM diagram or an informal description of the TM at an appropriate level of detail. Comment your code to make clear what is intended.

8. Diagonalization

In this problem, you are asked to prove that the "diagonal language"

$D = \{ <M> | <M> \notin L(M) \}$

cannot be a Turing-recognizable language. (This is the language consisting of all TMs that do not recognize themselves!)

(a) Proof by contradiction. Assume D is Turing-recognizable. Then there must be some particular TM that recognizes D. Call it $M*$. Write down in symbols what the foregoing fact means in terms of $L(M)$ and $<M>$ (this part has been done for you.)

$$D = L(M*) = \{ <M> | <M> \notin L(M) \}$$

(b) Using the definition of D , write out exactly what is in the set $L(M*)$:

$$L(M*) = \{ <M> |$$

(c) Is $<M*>$ a member of $L(M*)$? Don't answer yet. Let us explore this. First suppose that $<M*> \in L(M*)$. What can we then conclude from the definition of $L(M*)$ in part (b)?

(d) Next suppose that $<M*> \notin L(M*)$. What can we now conclude from the definition of $L(M*)$ in part (b)?

(e) Parts (c) and (d) together form a blatant contradiction. Explain why.

(f) Given that a contradiction has been derived, what assumption that we made earlier *must* be false?

(g) What is the overall conclusion that can be derived from this *reductio ad absurdum* argument?

9. State Diagram

Construct the state diagram of a DFA that accepts

$$L = (00 + 1)^* \cap ((000)^*(11)^*)$$

Explain each step in your construction in a short phrase that allows the reader to understand how your construction works, and why it is correct.

10. Decision Problem

Is there an algorithm to decide, given a Turing-recognizable set L, whether or not $L \cap (01)^* \neq \emptyset$? If so describe the algorithm. If not, prove there is no such algorithm.