Algorithms CSs545 — Homework #7
Review for the exam – optional

December 17, 2002

1. Let $G(A \cup B, E)$ be a bipartite graph. The greedy algorithm for finding a maximal matching $M \subseteq E$, works as follows: It repeatedly find and edge $(u, v) \in E$ such that neither u nor v are adjacent to edges in M, and add (u, v) to M. Let M denote the matching found by this algorithm, and let M^* be an optimal matching in G (i.e. matching with maximal cardinality).

   - Given an example of G (with arbitrary number of vertices) for which $2|M| = |M^*|$.
   - Prove that at every graph G, $\quad 2|M| \geq |M^*|$. (Hint — show that every edge of M can prevent at most 2 edges of M^* to exist.)
   - How would you implement the greedy algorithm, so it would run in time $O(|A \cup B| + |E|)$.
   - Show that there are $|M^*| - |M|$ vertex-disjoint augment path, for every matching $M \subseteq E$. (vertex disjoint means that no two vertices share a vertex). Hint: Start from every vertex $a \in A$ which is in M^*. Start constructing alternating path. How many of these paths are augmenting path ?
   - Assume that all augment paths in the graph have length at least t (for some parameter $t > 3$). What can you say about $\frac{|M|}{|M^*|}$? Use the previous section

2. Let $S = \{(x_1, y_1), \ldots (x_n, y_n)\}$ denote a set of points given to you, where $x_1 < x_2 < \ldots x_n$. Also given that $y_i = 1$ or $y_i = 0$ for every i.

   The question is to find a is collection of paths in the plane, of minimal total length, so that it is possible to get from every point of S to every other point by following one of theses paths. These paths also need to be built using only horizontal and vertical edges, and endpoins of these edges can lie only one the lines $y = 0$ and $y = 1$. Your algorithm should run in $O(n)$.

Hint: Scan the points of S from left to right. Using a simple dynamic programming, where two values are maintained at each iteration.