CSc 573 **Homework 4** **Spring 2007**

This homework is due Thursday April 19 at the start of class. Questions are drawn from the material in Chapters 8 of the text on space complexity.

The homework is worth 50 points. When questions with several parts do not specify the points for each part, each part has equal weight.

Remember to write on just one side of a page, do not use scrap paper, put your answers in the correct order, and staple your pages together. If you can't solve a problem, state this, and write only what you know to be correct. Neatness and conciseness count.

(1) **(Punched card game)** (15 points) Recall the language PunchedCards from Homework 3. In this decision problem, there is a deck of cards, with holes punched in two columns, and the question is whether there is a way to flip the cards vertically so that when they are stacked on top of each other, no hole is punched all the way through.

Suppose we define a *Punched Card Game* that has two players, I and II, which each have a set of n cards, and that alternately take turns stacking one flipped card on top of another. Player I moves first, and wins if after all the cards are stacked, no hole is punched all the way through; Player II wins if some hole is punched through.

Prove that the language PunchedCardGame, corresponding to instances of this game is which Player I has a winning strategy, is PSpace-complete.

(Hint: Use the correspondence developed in Homework 3 between deciding the language PunchedCards and Satisfiability, and exploit the similarity of the Punched Card Game to the Formula Game defined in the book.)

(2) **(Linear bounded automata)** (10 points) Consider the language

$$\text{LBA} := \left\{ \langle M, w \rangle : \begin{pmatrix} M \text{ is a decider that runs in } O(n) \text{ space} \\ \text{and accepts input } w \end{pmatrix} \right\}.$$

Prove that LBA is PSpace-complete.

(Hint: Use the fact that TQBF is PSpace-complete, so every language in PSpace can be reduced in polynomial time to TQBF, and that there is a decider for TQBF that runs in linear space.)

(3) **(Cat and mouse game)** (15 points) Prove that the Cat and Mouse Game defined in the book in Problem 8.15 is in $\mathcal{P}$.

(Hint: To show that whether the cat has a winning strategy for an instance of the Cat and Mouse Game can be decided in polynomial time, design an algorithm that iteratively computes the set of game configuration for the instance in which the cat wins. Initially this set consists of configurations in which the cat has pounced on the mouse. This set can be enlarged by repeatedly finding all those configurations that can reach a known winning configuration in one move.)

(4) **(Balanced parentheses)** (10 points) Consider the language Bal of balanced parentheses. Prove that Bal $\in \mathcal{L}$.

(Hint: Show that whether a string of left- and-right parenthesis is balanced can be decided using a constant number of counters.)