

Name _____

Instructions This exam consists of three problems worth a total of 50 points.

Do Problem (1), and then *choose two questions* from among Problems (2) through (4). In other words, you will do a total of three questions. If you do more than three questions, only three will be graded.

You have 75 minutes. The exam is closed book, closed notes, and closed calculator. Good luck!

- (1) **(Short answer)** (10 points) Give a short answer to each of the following questions.
- (a) (2 points) Define the complexity class \mathcal{NP} .
- (b) (2 points) Define what it means to say there is a *polynomial time reduction* of problem X to problem Y .
- (c) (2 points) Define what it means to say that problem X is *NP-complete*.
- (d) (4 points) What is the *practical import* of showing that problem X is NP-complete?

- (2) **(Hamiltonian cycle)** (20 points) In an undirected graph G , a *Hamiltonian cycle* is a cycle that visits every vertex of G exactly once. Let

$$\text{UndirHamCycle} := \left\{ \langle G \rangle : \left(G \text{ is an undirected graph that} \right. \right. \\ \left. \left. \text{contains a Hamiltonian cycle} \right) \right\}.$$

Prove that UndirHamCycle is NP-complete.

(Hint: Use a reduction from the Undirected Hamiltonian Path Problem, which is to decide the language

$$\text{UndirHamPath} := \left\{ \langle G, s, t \rangle : \left(G \text{ is an undirected graph that} \right. \right. \\ \left. \left. \text{contains a Hamiltonian } (s, t)\text{-path} \right) \right\}.$$

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- (3) (**Sovereign set**) (20 points) In an undirected graph $G = (V, E)$, a *sovereign set* is a vertex subset $S \subseteq V$ such every edge in E has at most one endpoint in S . Let

$$\text{SovereignSet} := \left\{ \langle G, k \rangle : \left(G \text{ is an undirected graph that contains} \right. \right. \\ \left. \left. \text{a sovereign set of at least } k \text{ vertices} \right) \right\}.$$

Prove that SovereignSet is NP-complete.

(Hint: Use a reduction from the Clique Problem, which is to decide the language

$$\text{Clique} := \left\{ \langle G, k \rangle : \left(G \text{ is an undirected graph that} \right. \right. \\ \left. \left. \text{contains a clique of size at least } k \right) \right\}.$$

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- (4) **(Half satisfiability)** (20 points) A Boolean formula F in conjunctive normal form is *half-satisfiable* if it contains a truth assignment to its variables such that exactly half of its clauses evaluate to true, and exactly half of its clauses evaluate to false. Let

$$\text{HalfSat} := \left\{ \langle F \rangle : \left(\begin{array}{l} F \text{ is a Boolean formula in 3-conjunctive} \\ \text{normal form that is half satisfiable} \end{array} \right) \right\}.$$

Prove that HalfSat is NP-complete.

(Hint: Use a reduction from the Three-Satisfiability Problem, which is to decide the language

$$\text{ThreeSat} := \left\{ \langle F \rangle : \left(\begin{array}{l} F \text{ is a Boolean formula in 3-conjunctive} \\ \text{normal form that is satisfiable} \end{array} \right) \right\}.$$

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