CSc 573 | **In-class Exam 2** | **Spring 2007**

Name ____

Instructions This exam consists of three problems worth a total of 50 points.

You must do Problem (1). Choose *one* of Problems (2) or (3). You must do Problem (4). In other words, you will do a total of three questions. If you do more than three questions, only three will be graded.

You have 75 minutes. The exam is closed book, closed notes, and closed calculator. Good luck!

(1) **(Short answer)** (10 points) Give a short answer to each of the following questions.

(a) (2 points) Define the complexity class $\mathcal{NL}$.

(b) (2 points) Define what it means to say there is a *logarithmic-space reduction* of problem X to problem Y.

(c) (3 points) Define what it means to say that problem X is *PSpace-complete*.

(d) (3 points) What does *Savitch's Theorem* state?

(2) **(Equal-size sum-split)** (20 points) Given a multiset $\mathcal{S} = \{x_1, x_2, \ldots, x_n\}$ of integers, an *equal-size sum-split* of $\mathcal{S}$ is a partition of $\mathcal{S}$ into two multisubsets $\mathcal{A}$ and $\mathcal{B}$ such that (1) $\mathcal{A} \cap \mathcal{B} = \emptyset$, (2) $\mathcal{A} \cup \mathcal{B} = \mathcal{S}$, (3) $|\mathcal{A}| = |\mathcal{B}|$, and

$$\sum_{a \in \mathcal{A}} a \;=\; \sum_{b \in \mathcal{B}} b.$$

Let

$$\text{EqualSizeSumSplit} \;:=\; \left\{ \langle \mathcal{S} \rangle \;:\; \begin{pmatrix} \mathcal{S} \text{ is a multiset of integers that} \\ \text{has an equal-size sum-split} \end{pmatrix} \right\}.$$

Prove that EqualSizeSumSplit is NP-complete.

(Hint: Use a reduction from the Subset Sum Problem, which is to decide the language

$$\text{SubsetSum} \;:=\; \left\{ \langle \mathcal{S}, k \rangle \;:\; \begin{pmatrix} \mathcal{S} \text{ is a multiset of integers that contains} \\ \text{a multisubset } \mathcal{T} \text{ such that } k = \sum_{t \in \mathcal{T}} \end{pmatrix} \right\}.$$

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(3) **(Two-consecutive ones)** (20 points) A matrix M whose entries have the values 0 or 1 has the *two-consecutive-ones property* iff (1) all of its rows have at most two ones, and (2) its columns can be reordered such that in the reordered matrix $\widetilde{M}$, in every row of $\widetilde{M}$ that has two ones, these ones are adjacent. Let

$$\text{TwoConsecOnes} := \left\{ \langle M, k \rangle : \begin{pmatrix} M \text{ is a matrix of 0's and 1's such that after changing} \\ \text{at most } k \text{ ones to zeroes, the resulting matrix has the} \\ \text{two-consecutive-ones property} \end{pmatrix} \right\}.$$

Prove that TwoConecOnes is NP-complete.

(Hint: Use a reduction from the Undirected Hamiltonian Path Problem, which is to decide the language

$$\text{UndirHamPath} := \left\{ \langle G, s, t \rangle : \begin{pmatrix} G \text{ is an undirected graph that} \\ \text{contains a Hamiltonian } (s,t)\text{-path} \end{pmatrix} \right\}.$$

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(4) **(Inequivalent DFAs)** (20 points) Let

$$\text{NotEquivDFAs} := \Big\{\langle M_1, M_2\rangle \;:\; M_1, M_2 \text{ are DFAs with } L(M_1) \neq L(M_2)\Big\}.$$

Prove that NotEquivDFAs is a member of $\mathcal{NL}$.

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