DUE: Tuesday 4 March

Midterm Exam: Thursday 6 March, in class

Problems

1. Write-Once Memory

Say that a *write-once TM* is a standard one-tape TM that can alter each tape square *at most once*—including the input portion of the tape.

Show that this variant TM model is equivalent to the standard TM model.

HINT: As a first step, show how to simulate a standard TM by a TM that can alter each tape square *at most twice*. Then simulate this by the write-once model. Use *lots* of tape.

2. Pairing Functions

Homer & Selman, Homework 3.2

3. $\neg\exists$ Size Optimization Algorithm

Homer & Selman, Homework 3.3

4. Orderly Enumeration

Show that a set L is decidable iff some TM enumerates L *in increasing numerical order* (same thing as increasing *lexicographic order* because of the 2-adic number representation). Thus we see that c.e. but undecidable sets can be enumerated—but only in some jumbled order!

This problem is the same as Homer & Selman, Homework 3.4

5. Hard Sets

Show that $L_U = \{(e,w) \mid w \in L(M_e)\}$, $K = \{e \mid \phi_e(e) \downarrow\}$ and $L_{NES} = \{e \mid L(M_e) \neq \emptyset\}$ are all $\equiv_m$-equivalent, and hence *all are complete* in **CE**.

6. Complementary Hard Sets

Show that $\overline{L_U}$, $\overline{K}$ and $L_{ES} = \{e \mid L(M_e) = \emptyset\}$ are all $\equiv_m$-equivalent, and hence *all are complete* in **co**$-$**CE**. *HINT:* This problem should not require you to construct reductions, as in the previous problem. Use the previous problem and properties of $\leq_m$ reduction to solve this one.

7. Machine-dependent Reduction

Consider the problem of testing whether a 2-tape deterministic TM ever writes a non-blank symbol on its second tape. Formulate this problem as a language, and show that it is c.e. but not decidable.