DUE: Tue 8 April 2008

Reading

Check course web page for reading assignments.

Problems

1. *LOOP* $\rightarrow$ **Primitive Computable**

Prove that every function computed by a *LOOP* program is primitive computable. *HINT:* Use induction on the structure of a *LOOP* program. Be sure to allow for both loop nesting (which increases depth) *and* concatenation (;) of loop programs of the same or different depths. It will be technically convenient to allow *LOOP* programs to take multiple arguments (have input set I larger than 1) and to return multiple arguments (have output set O larger than 1).

2. Primitive Computable $\rightarrow$ *LOOP*

Prove that every primitive computable 5unction is computed by a *LOOP* program. *HINT:* Every member of **Pr** has a signature.

3. Non-Repeating Enumeration

Prove that a language L is computably enumerable if and only if there is a TM that enumerates L *without ever repeating* an element of the language.

4. A Reduction

Text, Homework 3.26. Since $11 \in L$ is a property of languages, Rice's Theorem says that this question is undecidable. However, this question asks you to provide the details of a specific reduction that shows that this problem is undecidable.

Call the language that is to be shown undecidable L_{11}. This language is the index set $L_{11} = \{e | \, 11 \in L(M_e)\}$. You are asked to actually perform the reduction in detail. Denote a (hypothetical) decider for L_{11} as M_{11}.

(a) State the reduction $X \leq_m L_{11}$ that you plan to use, and sketch the diagram of the reduction. Show a dotted box that is a decider for X, containing a solid box for M_{11}). Diagram the inputs to each box and their types (string, character, etc.)

(b) Show by diagram the TM that results from applying the reduction function c to the inputs to the dotted box for X. This TM will be called $M_{c(\,\ldots\,)}$.

(c) State the property that $M_{c(\,\ldots\,)}$ exhibits that relates it to the decision question for X. In other words, show that $w \in X \Leftrightarrow c(\,\cdots\,) \in L(M_{11})$.

(d) Complete the argument that $X \leq_m L_{11}$, using parts (a) and (c).

5. Undecidable or Decidable?

Consider the following languages. All of them are "machine-dependent", so that Rice's Theorem will not apply. For each language, say whether it is decidable or undecidable, and prove your assertion.

(a) $L_a = \{ (e, w, p) \mid M_e$ reaches state p on input w starting from $q_0 \}$

(b) $L_b = \{ (e, p) \mid M_e$ has a configuration $[\alpha q \beta]$ yielding, in one or more steps, a configuration containing state $p\}$

(c) $L_c = \{ (e, \mathbf{a}) \mid M_e$ writes character **a** when started on the empty tape $\}$

(d) $L_d = \{ e \mid M_e$ writes a non-blank character when started on the empty tape$\}$

(e) $L_e = \{ (e, w) \mid M_e$ eventually moves its head left when started on $w\}$

(f) $L_f = \{ (e, w) \mid (\exists k)\ M_e$ visits *at most* k cells on input w }. *NOTE:* The language $L = \{ (e, w, k) \mid M_e$ visits *at most* k cells on input w } is **decidable** (Why?). The current problem asks what happens when you prefix this predicate by $\exists k$, so that k is not an input parameter.

6. Egotistical Interpreter

Show that there is a partial computable function ϕ_n such that

$$\forall y\ \phi_n(y) = \phi_y(n)\ .$$

Thus program n when given an input y, mimics program y when run on n's own name.

7. "Decidable" decidable?

Is it decidable whether or not a given language is decidable? That is, given a TM M_e, is it decidable whether or not $L(M_e)$ is decidable?

(a) Precisely define, in terms of an index set, the question at issue in (a). *HINT:* $L_{decidable} = \{ e \mid \cdots \}$.

(b) State your finding on whether or not the language (index set) $L_{decidable}$ is decidable or undecidable.

(c) Prove your assertion in (b).