Theory of Computaion

Lecture 06A

More Recursion Theorem

The Recursion Theorem: Applications

- Example: Self-Printing Program $\phi_n(x) = n$
- For a specific general-purpose programming language

```
char *s="char *s=%c%s%c;%cmain(){printf(s,34,s,34,10,10);}%c";
main(){printf(s,34,s,34,10,10);}
```

- 34 = ascii " 10 = ascii \n
- Printf statement says: take the string s and print it after inserting a quoted copy of itself

The Recursion Theorem: Applications

- Another proof of Rice's Theorem for <u>functions</u>
- A "property of the p.c.f.s" is a predicate P on the natural numbers such that

$$\phi_i = \phi_j \Rightarrow P(i) = P(j)$$

- P is a property of the functions (behavior), not a property of the indices (programs).
- P is nontrivial iff neither always true or always false
- Thm: every nontrivial property of the p.c.f.s is undecidable.
- Pf: Suppose P is such a property. Because non-trivial, there are indices a and b such that $P(a)=0$ $P(b)=1$. Suppose P were decidable.

The Recursion Theorem: Applications

- Then the function
  $$\psi(x) = if\ P(x) = 1\ then\ a\ else\ b$$
  would be a t.c.f.
- Now for all x
  $$P(\psi(x)) = if\ P(x) = 1\ then\ P(a)\ else\ P(b)$$
  $$= if\ P(x) = 1\ then\ 0\ else\ 1$$
- So we have that
  $$\forall x\ P(\psi(x)) \neq P(x).$$
- Which implies, since P is a property of functions:
  $$\forall x\ \phi_{\psi(x)} \neq \phi_x.$$
- Since ψ is a t.c.f., this last contradicts the Recursion Theorem, since it says "ψ has no fixed point"