Pantactic Squares

The material that follows is based on a paper in a mathematical journal and a Web page [1-2].

A 5×5 pantactic square is a 5×5 two-color grid pattern in which every 2×2 subpattern is different.

There are $2^4 = 16$ 2×2 patterns:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
</tr>
</tbody>
</table>

The identifying labels are obtained by following the rows left to right, taking white cells to be 0 and black cells to be 1. The resulting binary number is converted to hexadecimal.

Here is an example of a 5×5 pantactic square:

The 2×2 subpatterns from left to right, top to bottom are: DB247E81FA05C936. This string uniquely identifies this pantactic square, although there are more concise ways of doing this.

Although there are $2^{25} = 33,554,432$ 5×5 two-color grid patterns, there are only 800 essentially different 5×5 pantactic squares. These 800 squares can be grouped into 16 categories according to common structural properties.

Properties of 5×5 Pantactic Squares

5×5 pantactic squares have some surprising properties, especially lack of symmetry. While many patterns are the same after some kind of rotation or reflection, 5×5 pantactic squares are not: No combination of rotations, reflection, (horizontal, vertical, or diagonal), or color inversion of a 5×5 pantactic square produces the same 5×5 pantactic square.

Another property of 5×5 pantactic squares is a limitation of connected paths they can contain. A connected path in a grid pattern is a sequence of cells of the same color, all of which share an edge. For example, in
the black cells form a connected path, while in

they do not, because in two places they connect only at corners.

A 5×5 pantactic square cannot have a connected path that reaches from one edge to the opposite one.

A connected block is a collection of cells of the same color in which the cells share edges. An example of a connected block is

The maximum number of connected cells in a connected block in a 5×5 pantactic square is eight.

There are 50 essentially different basic blocks in 5×5 pantactic squares. Here are four of them:

Pantactic Patterns

Some basic blocks can be arranged in ways that form repeating patterns of arbitrarily large size, all of whose 5×5 subpatterns are pantactic squares. Two examples are:
The arrangement here is a regular tiling.

The arrangement here is a tiling with a three-row offset.

**Generalizations**

There are other questions to be addressed. The literature on pantactic squares seems to be limited to $5 \times 5$ ones. Are there larger ones? Are there non-square pantactic designs? What about more than two colors?