Signature Sequences

An interesting class of fractal sequences consists of signature sequences for irrational numbers. The signature sequence of the irrational number $x$ is obtained by putting the numbers

$$i + j \times x \quad i, j = 1, 2, 3, \ldots$$

in increasing order. Then the values of $i$ for these numbers form the signature sequence for $x$, which is denoted by $\mathcal{S}(x)$.

Here’s the signature sequence for $\phi$, the golden mean:

1, 2, 1, 3, 2, 4, 1, 3, 5, 2, 4, 1, 6, 3, 5, 2, 7, 4, 1, 6, 3, 8,
5, 2, 7, 4, 9, 1, 6, 3, 8, 5, 10, 2, 7, 4, 9, 1, 6, 11, …

Both upper trimming and lower trimming of a signature sequence leave the sequence unchanged.

Signature sequences have a characteristic appearance, but they vary considerably in detail depending of the value of $x$.

Signature sequences start with a run $1, 2, \ldots, n+1$, where $n = \lfloor x \rfloor$, the integer part of $x$. The larger the value of $x$, the more quickly terms in the sequence get larger. Most signature sequences display runs, either upward or downward or both — which one is usually a matter of visual interpretation. At some point, most signature sequences become interleaved runs. This sometimes gives the illusion of curves.

Although signature sequences are defined only for irrational numbers, the algorithm works just as well for rational numbers. Although signature sequences for rational numbers are not fractal sequences, they are as close as you could determine manually. The structure of a signature sequence depends on the magnitude of $x$. Furthermore, there are irrational numbers arbitrarily close to any rational number. There is no difference in the initial terms of signature sequences for numbers that are close together. For example, $\mathcal{S}(3.0)$ and $\mathcal{S}(\pi)$ do not differ until their 117th terms.

It’s also worth noting that there really is no way, in general, to perform exact computations for irrational numbers. Computers approximate real numbers, and hence irrational numbers, using floating-point arithmetic. A floating-point number representing an irrational number is just a (very good) rational approximation to the irrational number. For example, the standard 64-bit floating-point encoding for $\pi$ is
Figure Ω.1 shows grid plots for some signature sequences. Signature sequences for large numbers are not included because they are unwieldy.

**Using Signature Sequences in Weaving Drafts**

Signature sequences can be used as the basis for threading and treadling sequences. To use signature sequences for this purpose, it is necessary to bring the values of terms within the bounds of the number of shafts and treadles used. The mathematically reasonable way is to take their residues, modulo the number of shafts or treadles, using 1-based arithmetic. Figure Ω.2 shows residue sequences derived from signature sequences. In most cases, taking residues preserves the essential characteristics of signature sequences.

Sequences like these, if used directly, produce drawdown patterns that lack repeats or symmetry. More attractive patterns can be obtained by taking a small portion of a signature sequence and then reflecting it to get symmetric repeats.

Figure Ω.3 shows a draft for such a sequence with 16 shafts and treadles and a \( \frac{2}{2} \) twill tie-up.

It seems natural to use initial terms of a signature sequence. The structure of signature sequences, however, changes as the sequence goes on. Figure Ω.4 shows magnified portions of the drawdown pattern for a signature sequence. This suggests that it might be worth trying subsequences of signature sequences in various locations.

Figure Ω.5 shows some drawdown patterns for various combinations of signature sequences. All have 16 shafts and treadles and \( \frac{2}{2} \) twill tie-ups.
Figure 1
Grid Plots for Signature Sequences
Figure Ω.1, continued. Grid Plots for Signature Sequences
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Figure $\Omega$.2. Signature Sequence Residues
Figure Ω.3. Drawdown for A Reflected Portion of $S(\pi)$
Figure Ω4. Magnified Portions of the $\phi \times \phi$ Signature Drawdown Plane
Figure 2.5. Drawdown Patterns for Signature Sequences
Figure Ω 5, continued. Drawdown Patterns for Signature Sequences

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threading: \( \pi \), terms 61-120

threading: \( e \), terms 1-60

threading: \( e \), terms 1-60

threading: 0.9, terms 61-120
Figure Ω.5, continued. Drawdown Patterns for Signature Sequences