Maximal Color Patterns

The last section described how to determine if a color pattern is weaveable and, if so, how to create a draft for it.

This section looks at weaveable color patterns from a different perspective: How to create color patterns that are guaranteed to be weaveable and have as many colors as are possible.

In this context a color pattern is an \( i \times j \) array of colored cells. An array in which every column and row is labeled with a different color is called \textit{maximal}.

One question is how many cells are needed to create a weaveable pattern that has \( k \) different colors. Obviously, this can be done with a \( 1 \times k \) or \( k \times 1 \) pattern: a single row or column with a cell for each different color. These cases, however, are degenerate and uninteresting.

For maximal patterns, \( k \) is partitioned into two parts. There are \( k - 2 \) non-degenerate size combinations, given by

\[
i = k - j \quad 2 \leq j \leq k - 2
\]

Since these arrays have \( i \times j \) cells, the largest array occurs for \( i = j \) or \( i = j \pm 1 \), depending on whether \( k \) is even or odd.

Suppose there are eight colors and a \( 4 \times 4 \) array as shown in Figure \( \Omega.1 \).

\[
\begin{array}{cccc}
A & B & C & D \\
E & & & \\
F & & & \\
G & & & \\
H & & & \\
\end{array}
\]

Figure \( \Omega.1 \). A \( 4 \times 4 \) Array

It is obvious that it’s possible to have all \( k \) colors in such an array. Figure \( \Omega.2 \) shows one such pattern.
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The remaining cells in Figure Ω.2 can be colored in any of the ways the column and row labels allow. Since there are eight cells of unspecified color, there are $2^8 = 256$ possible patterns based on Figure 2.

For $k$ a multiple of four and $i = j = k / 2$, it is possible to assign colors to cells so that each color occurs $k / 4$ times ($i \times j = k^2 / 4$). Here is a coloring algorithm for constructing such color-balanced patterns:

- For each odd-numbered row, assign alternate cells the column and row colors.
- For each even-numbered row, assign alternate cells the row and column colors.

Figure Ω.3 shows the result for a $4 \times 4$ array.

For other array shapes, color balance is not possible, but the coloring
algorithm given above assures $k$-colored patterns. The patterns produced by this algorithm can be quite attractive. See Figure Ω.4 for an example.

![Algorithmic Pattern](image)

Figure Ω.4. An Algorithmic Pattern

**Transformations that Preserve Weaveability**

Given a weaveable color pattern, there are several kinds of changes that can be made to it that preserve Weaveability:

1. duplicating existing rows and columns
2. deleting rows and columns
3. rearranging rows and columns
4. rotating the pattern in 90° increments
5. flipping the pattern horizontally, vertically, or diagonally
6. adding solid-colored rows and columns

These changes do not require knowledge of the colors assigned to columns and rows. Here are two that do:

7. adding a column whose cells are colored either with the new column color or their corresponding row colors, and similarly for rows
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8. setting the color of a cell to the color of its column or row

The first kind of change, duplicating existing rows and columns, offers many design possibilities. For example, duplicating adjacent rows and columns can be used to produce bands of any desired width. Mirroring, horizontal, vertical, or both also follows. Figure 5 shows a weavable color pattern created using only duplications of the rows and columns of an algorithmic pattern:

Figure Ω5. A Weavable Color Pattern