A Network Calculus for Multi-Hop Fading Channels

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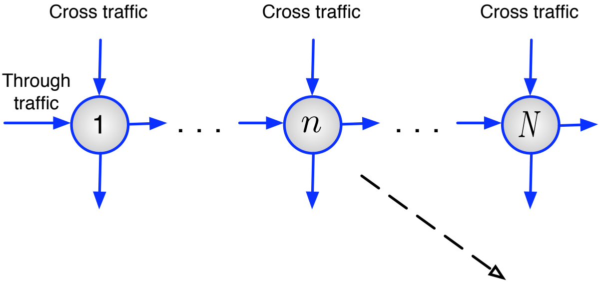
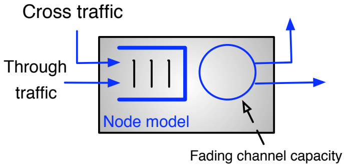
Performance Analysis of Multihop Wireless Network



- Intermediate nodes are store and forward relays
- A fading channel is characterised by its channel capacity



Fading Channel Capacity

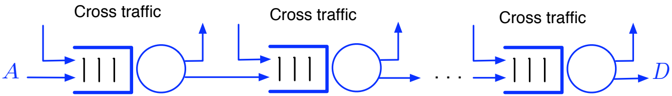
- Channel capacity [Shannon 1948]

$$C(\gamma) = W \log(1 + \gamma)$$

- $\gamma = \bar{\gamma}|h|^2$ for fading channels
- Channel gain h is a complex r.v.

Q: How do fading channel properties affect multihop network performance?

Network Model



- Fluid-flow traffic, discrete time
- Arrival and service are independent
- I.i.d. cross traffic at each node
- Time-varying random service that is equal to the Instantaneous channel capacity

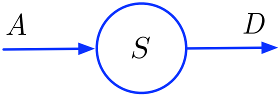
$$C(\gamma_t) = W \log\left(g(\gamma_t)\right), \quad \gamma_t = \bar{\gamma}|h_t|^2$$

- Computing this service distribution is **hard!**

Related Work: Multihop network performance analysis

- Simplified channel models
  - FSMC model [Wang and Moayeri 1995][Sadeghi et al 2008]
    - more than two states models may not be tractable
    - not easily extended to multihop networks
  - ON-OFF model
    - tractable but very simplified model
  - used in queuing theory [Ishizaki 2007], network calculus [Ciucu 2011], effective bandwidth [Hasan,Krunz,Matta 2004]
- Effective capacity [Wu and Negi 2003]
  - log-MGF of the channel capacity
  - tractable only for low SNR where $\log(1+\gamma) \simeq \gamma$
- Physical layer models [Hasna and Alouini 2003]
  - outage probability for AF wireless relay network
  - expression for MGF of end-to-end SNR
  - not suitable for network analysis

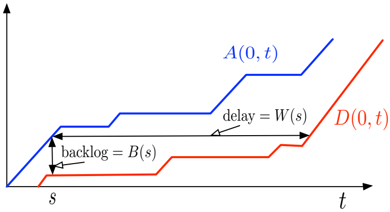
Network Calculus



- $(\min, +)$ dioid algebra
- Backlog: $B(s) = A(0,s) - D(0,s)$
- Delay: $W(s) = \inf\{u \geq 0 : A(0,s) \leq D(0,s+u)\}$

- Dynamic server [Chang 2000]

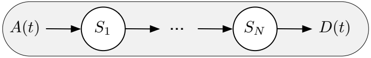
$$D(0,t) \geq \inf_{u \leq t}\{A(0,u) + S(u,t)\}$$
$$= A * S(0,t)$$



- Network service:

$$S_{\text{net}}(\tau,t) = S_1 * S_2 * \cdots * S_N(\tau,t)$$

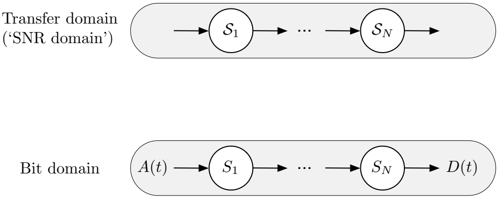
Network Analysis in Bit Domain



Bit domain

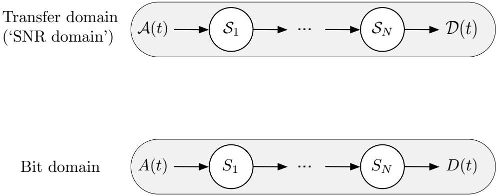
- Arrivals and departures are measured in bits
- For fading channels, service is given in terms of $\log(g(\gamma_t))$
- Distribution of S is not easy to work with

SNR Domain



- Service in terms of $g(\gamma_t)$ rather than $\log(g(\gamma_t))$ – more tractable
- SNR service $\mathcal{S}(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$ resides in the <u>SNR domain</u>

SNR Domain



- Service in terms of $g(\gamma_t)$ rather than $\log(g(\gamma_t))$ – more tractable
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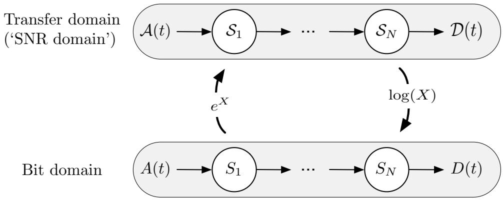
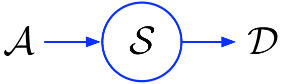
Our Approach



- SNR domain is governed by $(\min, \times)$ dioid algebra
- Network SNR server

$$\mathcal{S}_{\text{net}}(\tau, t) = \mathcal{S}_1 \otimes \mathcal{S}_2 \otimes \cdots \otimes \mathcal{S}_N(\tau, t)$$

(min, ×) Network Calculus



- Service: $\mathcal{S}(\tau, t) = \prod_{i=\tau}^{t-1} g(\gamma_i)$
- Arrival: $\mathcal{A}(\tau, t) = \prod_{i=\tau}^{t-1} e^{a_i}$
- Departure: $\mathcal{D}(0, t) \geq \mathcal{A} \otimes \mathcal{S}(\tau, t) = \inf_{\tau \leq u \leq t} \{\mathcal{A}(\tau, u) \cdot \mathcal{S}(u, t)\}$
- Backlog: $B(t) = \log \left(\frac{\mathcal{A}(0,t)}{\mathcal{D}(0,t)} \right)$
- Delay: $W(t) = \inf\{u \geq 0 : \mathcal{A}(0, t) \leq \mathcal{D}(0, t + u)\}$

Computation of $\mathcal{S}_1 \otimes \mathcal{S}_2$

- Mellin transform: $\mathcal{M}_X(s) = E[X^{s-1}]$
- For two independent servers

$$\mathcal{M}_{\mathcal{S}_1 \otimes \mathcal{S}_2}(s, \tau, t) \leq \sum_{u=\tau}^{t} \mathcal{M}_{\mathcal{S}_1}(s, \tau, u) \cdot \mathcal{M}_{\mathcal{S}_2}(s, u, t)$$

- For N i.i.d. fading channels

$$\mathcal{M}_{\mathcal{S}_{\text{net}}}(s, \tau, t) \leq \binom{N-1+t-\tau}{t-\tau} \cdot \left(\mathcal{M}_{g(\gamma)}(s)\right)^{t-\tau}, \quad \forall s < 1$$

- Moment bound: $Pr(X \geq a) \leq a^{-s}\mathcal{M}_X(1+s)$, $\forall a, s > 0$

Main Result: Statistical Performance Bounds

Define

$$\mathsf{M}(s,\tau,t) = \sum_{u=0}^{\min(\tau,t)} \mathcal{M}_{\mathcal{A}}(1+s,u,t)\cdot\mathcal{M}_{\mathcal{S}}(1-s,u,\tau)$$

- Backlog: $Pr\big(B(t) > b^{\varepsilon}\big) \le \varepsilon$, where

$$b^{\varepsilon} = \inf_{s>0}\left\{\frac{1}{s}\big(\log \mathsf{M}(s,t,t) - \log\varepsilon\big)\right\}$$

- Delay: $Pr\big(W(t) > w^{\varepsilon}\big) \le \varepsilon$, where

$$\inf_{s>0}\left\{\mathsf{M}(s,t+w^{\varepsilon},t)\right\} \le \varepsilon$$

Cascade of N i.i.d. Rayleigh Channels

- Service for Rayleigh channels
  - $g(\gamma) = 1 + \gamma = 1 + \bar{\gamma}|h|^2$
  - $|h| \sim$ Rayleigh r.v.
  - For i.i.d. Rayleigh fading channel

$$\mathcal{M}_{\mathcal{S}}(s, \tau, t) = \left(e^{1/\bar{\gamma}} \bar{\gamma}^{s-1} \Gamma(s, \bar{\gamma}^{-1}) \right)^{t-\tau}$$

- Arrivals: $(\sigma(s), \rho(s))$ bounded arrivals [Chang 2000]

$$\mathcal{M}_{\mathcal{A}}(s, \tau, t) \leq e^{(s-1)\cdot(\rho(s-1)\cdot(t-\tau)+\sigma(s-1))}, \quad s > 1$$

  - This traffic class includes Markov-modulated processes, effective bandwidth, etc.

Performance Bounds of N Rayleigh Channels

Define:

$$V(s) \triangleq e^{s\rho(s)} e^{1/\bar{\gamma}} \bar{\gamma}^{-s} \Gamma(1-s, \frac{1}{\bar{\gamma}})$$

- BACKLOG: $Pr\big(B(t) > b^{\varepsilon}_{\text{net}}\big) \leq \varepsilon$, where

$$b^{\varepsilon}_{\text{net}} = \inf_{s>0} \left\{ \sigma(s) - \frac{1}{s}\big(N \log(1 - V(s)) + \log \varepsilon\big) \right\}$$

- DELAY: $Pr\big(W(t) > w^{\varepsilon}\big) \leq \varepsilon$, where

$$\inf_{s>0} \left\{ \frac{e^{s(-\rho(s) w^{\varepsilon} + \sigma(s))}}{(1 - V(s))^N} \cdot \min\left\{1, (V(s))^{w^{\varepsilon}} (w^{\varepsilon})^{N-1}\right\} \right\} \leq \varepsilon$$

Numerical Results for N Rayleigh Channels

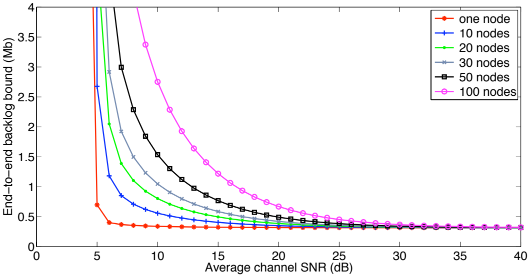
Model parameters

- $\Delta t = 1$ ms
- $W = 20$ kHz
- (σ, ρ) bounded traffic
- $\sigma = 50$ kb
- $\rho = 0$ to 60 kbps
- $\bar{\gamma} = 0$ to 40 dB
- $N = 1$ to 100

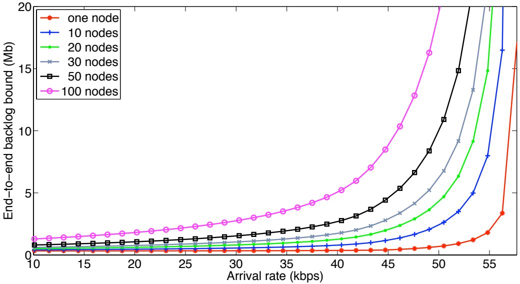
We used deterministically bounded traffic, hence, the only source of randomness is the fading channel!

Backlog Bounds for N Rayleigh Channels

- $b_{\text{net}}^{\varepsilon}$ vs. $\bar{\gamma}$
  - $\rho = 30$ kbps
  - $\varepsilon = 10^{-4}$



- $b_{\text{net}}^{\varepsilon}$ vs. ρ
  - $\bar{\gamma} = 10$ dB
  - $\varepsilon = 10^{-4}$

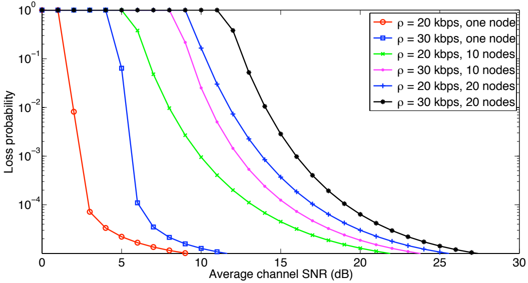


Backlog and Delays

(i) $\varepsilon(b)$ vs. $\bar{\gamma}$

- buffer size $= 400kb$

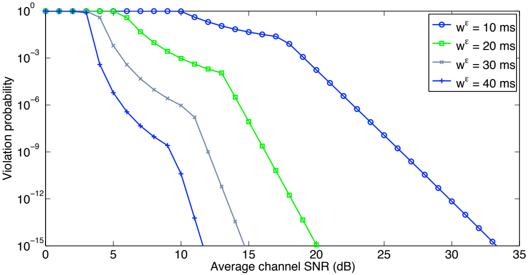
Waterfall curves for loss probability



(ii) $\varepsilon(w)$ vs. $\bar{\gamma}$

- $N = 10$
- $\rho = 20$ kbps

Tighter delay bounds at higher SNR



Conclusions

- New approach to analyze cascade of fading channels
- Analysis in SNR domain using $(\min, \times)$ dioid algebra
- Use Mellin transform and moment bound to compute end-to-end bounds
- Application to cascade of i.i.d. Rayleigh channels
  - Explicit bounds in terms of the physical channel parameters
  - Bounds scale linearly in N
- $(\min, \times)$ dioid algebra has potential applications in models with time varying channel models

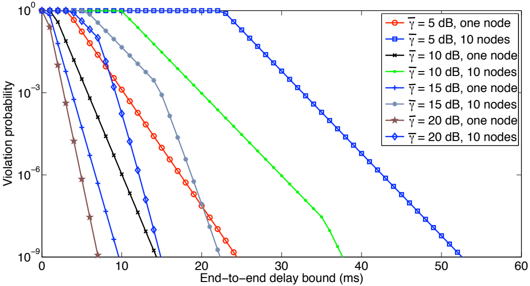
Thank you

Q & A

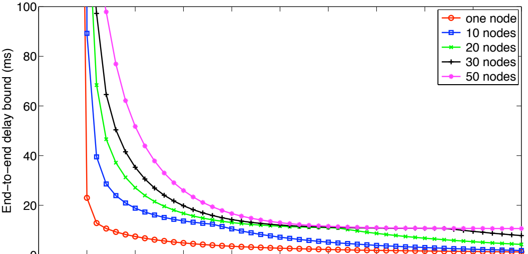
Delay bounds

(iii) $\varepsilon(w)$ vs. EtoE delay
– $\rho = 20$ kbps

– Effect of N on the violation prob. at low SNR is huge!



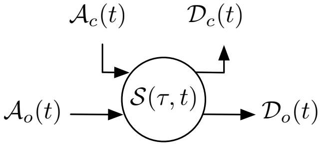
(iv) $w^{\varepsilon}_{\text{net}}$ vs. $\bar{\gamma}$
– $\rho = 30$ kbps
– $\varepsilon = 10^{-4}$



Fading Channels With Cross Traffic

- Leftover SNR service:

$$\mathcal{S}_o(\tau,t) = \frac{\mathcal{S}(\tau,t)}{\mathcal{A}_c(\tau,t)}$$



- Dynamic SNR server:

$$\mathcal{M}_{\mathcal{S}_o}(s,\tau,t) = \mathcal{M}_{\mathcal{S}/\mathcal{A}_c}(s,\tau,t) = \mathcal{M}_{\mathcal{S}}(s,\tau,t) \cdot \mathcal{M}_{\mathcal{A}_c}(2-s,\tau,t)$$

- N-node:

$$\mathcal{M}_{\mathcal{S}_{o,\text{net}}}(s,\tau,t) \le e^{(1-s)\cdot N\sigma_c(1-s)} \binom{N-1+t-\tau}{t-\tau} \cdot \left(\mathcal{M}_{g(\gamma)}(s) e^{(1-s)\cdot\rho_c(1-s)}\right)^{t-\tau}, \qquad s<1$$

Bounds of Rayleigh Channels With Cross Traffic

1. End-to-end Backlog of the through flow

$$b^{\varepsilon}_{o,\mathrm{net}}(t) \leq \inf_{s>0} \left\{ \sigma_o(s) + N\sigma_c(s) - \frac{1}{s}\left[N \log\left(1 - V_o(s)\right) + \log \varepsilon \right] \right\}$$

2. Delay bound, we estimate for $w^{\epsilon} \geq 0$

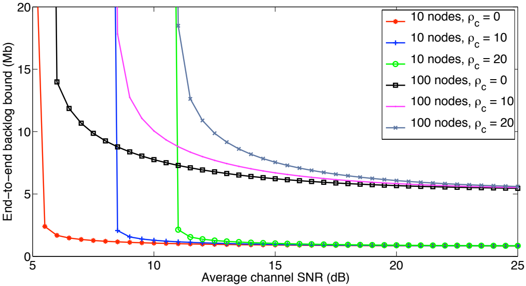
$$\inf_{s>0} \left\{ \frac{e^{s(-\rho_o(s)w + \sigma_o(s) + N\sigma_c(s))}}{(1 - V_o(s))^N} \cdot \min\left\{1, (V_o(s))^{w^{\epsilon}} (w^{\epsilon})^{N-1}\right\} \right\} \leq \epsilon$$

where,

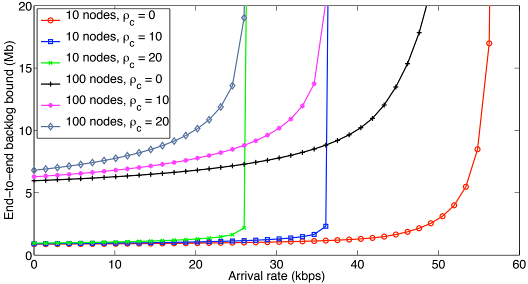
$$V_o(s) = e^{s\cdot(\rho_o(s) + \rho_c(s)} e^{1/\bar{\gamma}} \bar{\gamma}^{-s} \Gamma(1 - s, \bar{\gamma}^{-1})$$

Numerical results

- $\varepsilon = 10^{-4}$
- $W = 20$ kHz
- $\Delta t = 1$ msec.
- (σ, ρ) bounded through and cross traffic
- $\sigma_o = \sigma_c = 50$ kb



(i) $b^{\varepsilon}_{\mathrm{o,net}}$ vs. $\bar{\gamma}$
– $\rho_o = 30$ kbps



(ii) $b^{\varepsilon}_{\mathrm{o,net}}$ vs. ρ_o
– $\bar{\gamma} = 10$ dB

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