CSc 372

Comparative Programming Languages

26 : Prolog — Second-Order Predicates

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Second-Order Predicates

- When we ask a question in Prolog we will (if everything goes right) get an answer. **One** answer. We can if we want to ask Prolog to backtrack (using the semi-colon), but we will still only get one answer at a time.
- Furthermore, when we backtrack all the information gathered previously is lost.
- It isn't possible (in pure Prolog) to find the set of **all possible solutions** to a query.
- However, if we go outside pure Prolog (using the database manipulation features) we can construct procedures which collect all solutions to a query.
- They are called *second-order* because they deal with sets and the properties of sets, rather than about individual elements of sets.

Second-Order Predicates

- `setof(X,Goal,List)`
  - `List` is a collection of `X`s for which `Goal` is true.
  - `List` is sorted and contains no duplicates.
- `bagof(X,Goal,List)`
  - `List` is may contain duplicates.
- `setof` and `bagof` will fail if no `Goal`s succeed.
- `findall(X,Goal,List)`
  - `findall` will return `[]` if no `Goal`s succeed.

Examples

```
remove_duplicates(X, Y) :-
   setof(M, member(M,X), Y).

children(X,Kids) :-
   setof(C, father(X,C), Kids).
```

Uninstantiated Variables

- Consider `setof(X,Goal,List)` and `bagof(X,Goal,List)`.
- If there are uninstantiated variables in `Goal` which do not also appear in `X`, then a call to `setof` or `bagof` may backtrack, generating alternative values for `List`.
- If this is not the behavior you want, you can say

  `Y ^ Goal`

  meaning there exists a `Y` such that `Goal` is true, where `Y` is some Prolog term (usually, a variable).
- `findall` does this automatically.

Uninstantiated Variables. . .

- Consider this database:

```
foo(1,a).
foo(2,b).
foo(3,c).
```

- If we use both arguments of `foo` in our goal, we get what we expect:

```
| ?- findall(X/Y, foo(X,Y), L).
L = [1/a,2/b,3/c]
| ?- setof(X/Y, foo(X,Y), L).
L = [1/a,2/b,3/c]
| ?- bagof(X/Y, foo(X,Y), L).
L = [1/a,2/b,3/c]
```


Uninstantiated Variables. . .

- If we only use one of `foo`'s arguments in our goal, `findall` still gets us the expected result:

```
| ?- findall(X, foo(X,Y), L).
L = [1,2,3]
```

- But, `bagof` doesn't:

```
| ?- bagof(X, foo(X,Y), L).
L = [1]
Y = a ?  ;
L = [2]
Y = b ?  ;
L = [3]
Y = c
L = [1,2,3]
```


Uninstantiated Variables. . .

- So, instead we have to do:

```
| ?- bagof(X, Y^foo(X,Y), L).
L = [1,2,3]
```

SetOf — Drinkers

```
:- op(500, yfx, 'drinks').

john drinks whiskey.
martin drinks whiskey.
david drinks milk.
ben drinks milk.
helder drinks beer.
laurence drinks beer.
chris drinks coke.
louise drinks l_and_p.

?- setof(X, X drinks milk, S).
   X = _9109,
   S = [ben,david]
```


Implementing bagof

```
bagof(Item, Goal, _) :-
   assert(bag(marker)),
   Goal,
   assert(bag(Item)),
   fail.

bagof(_, _, Bag) :-
   retract(bag(Item)),
   collect(Item, [], Bag).

collect(marker, L, L).
collect(Item,ThisBag,FinalBag):-
   retract(bag(NextItem)),
   collect(NextItem,
      [Item|ThisBag], FinalBag).
```


Implementing setof

- `setof` is implemented as a call to `bagof` followed by a call to `sort` which puts the elements in order and removes duplicates.

Lee's Algorithm

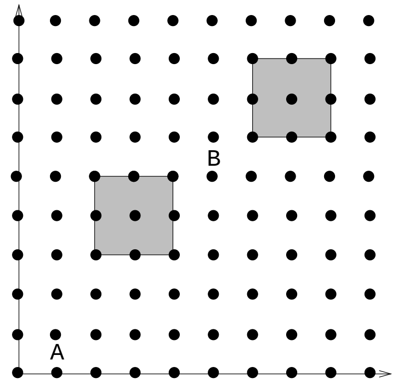
Lee's Algorithm

We are bext going to look a more involved example, an application from VLSI design. It uses the `setof` predicate to compute a shortest path between two points on a grid, subject to the conditions that

1. The path goes in the east-west-north-south direction only.
2. The path doesn't touch any obstacles.

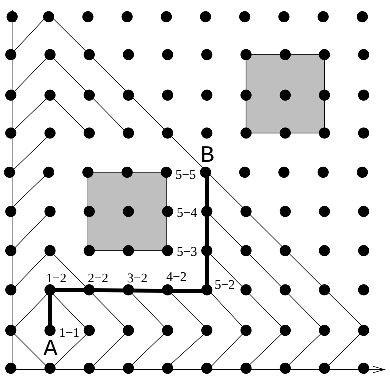
Lee's Algorithm

- VLSI routing on a grid.
- Find a shortest Manhattan route between A and B that doesn't pass through any obstacles.



Lee's Algorithm. . .

```
lee_route(A,B,Obstacles,Path) :-
   waves(B,[[A],[]],Obstacles,Waves),
   path(A,B,Waves,Path).
?- lee_route(1-1,5-5,[obst(2-3, 4-5),
      obst(6-6, 8-8)], P).
```



Lee's Algorithm. . .

Lee's algorithm works in two stages:

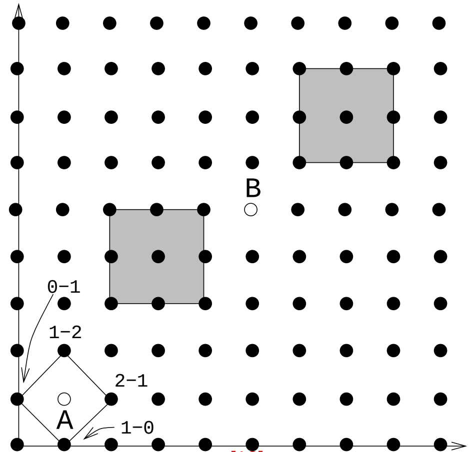
1. First we generate a sequence of waves, where the first wave consists of the starting point itself.
2. Then we use the set of waves to find a shortest path.

Lee's Algorithm…

- We start out with one wave which consists solely of the source point.
- From that point we generate all neighboring points. This forms the second wave.
- Each wave consists of points which are
  1. neighbors to points on the previous wave,
  2. not members of previous waves,
  3. not obstructed by any obstacles.
- We stop when the destination point is on the last generated wave.

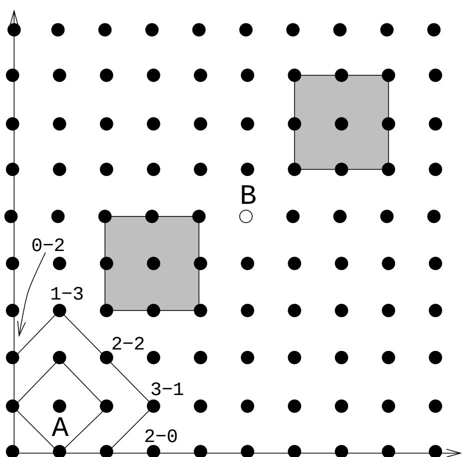
Lee's Algorithm…

```
LastW  =  []
Wave   =  [1-1]
NextW  =  [0-1,1-0,1-2,2-1]
```



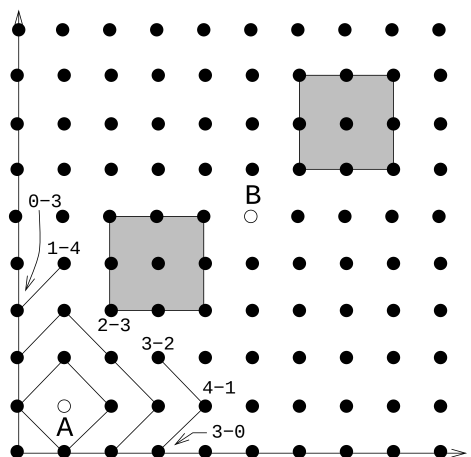
Lee's Algorithm…

```
LastW  =  [1-1]
Wave   =  [0-1,1-0,1-2,2-1]
NextW  =  [0-0,0-2,1-3,2-0,2-2,3-1]
```



Lee's Algorithm…

```
LastW  =  [0-1,1-0,1-2,2-1]
Wave   =  [0-0,0-2,1-3,2-0,2-2,3-1]
NextW  =  [0-3,1-4,3-0,3-2,4-1]
```



Lee's Algorithm. . .

```
aves(Destination,Wavessofar,Obstacles,Waves) :-
   Waves is a list of waves including
   Wavessofar (except, perhaps, it's last wave)
   that leads to Destination without crossing .
   Obstacles.

ext_waves(Wave,LastWave,Obstacles,NextWave) :-
   Nextwave is the set of admissible points
   from Wave, that is excluding points from
   Lastwave, Wave, and points under Obstacles.
```


Lee's Algorithm. . .

- The first `wave`-rule (the recursive base case for `wave`) states that once the last generated wave contains the destination point, we're done generating waves.
- The second `wave`-rule simply generates the next wave (using `next_wave`), and then adds it to the beginning of the list of waves. Note that the list of waves is a *list-of-lists*.

Lee's Algorithm. . .

- `next_wave` takes three input parameters:
  1. `Wave` is the last generated wave.
  2. `LastWave` is the wave generated before the last wave.
  3. `Obstacels` is the list of obstacles.
- `next_wave` uses `setof` to generate the set of all *admissible* points. A point is admissible if it belongs to the next wave.

Lee's Algorithm. . .

```
waves(B,[Wave|Waves],Obstacles,Waves) :-
   member(B,Wave), !.
waves(B,[Wave,LastWave|LastWaves],
      Obstacles,Waves) :-
   next_wave(Wave,LastWave,Obstacles,NextWave),
   waves(B,[NextWave,Wave,LastWave|LastWaves],
      Obstacles,Waves).

next_wave(Wave,LastWave,Obstacles,NextWave) :-
   setof(X,admissible(X,Wave,LastWave,Obstacles),
      NextWave).
```

Lee's Algorithm…

`X` is **adjacent** to the points on `Wave` (i.e. `X` is a point on the next wave) if

- `X` is a neighbor to a point `X1` on the previous wave (`Wave`, that is).
- `X` is not obstructed by an obstacle.

Lee's Algorithm…

Notice that `adjacent` uses a **generate-and-test** scheme:

1. `member` & `neighbor` work together to generate new possible points:
   (a) `member` generates points on the previous wave.
   (b) `neighbor` uses the points generated by `member` to generate points which are neighbors to the points on the last wave.
2. `obstructed` weeds out generated point that are under an obstacle.

Lee's Algorithm…

`X` is an admissible point if

1. it is a neighbor of a point on the previous wave
2. it is not on any previous wave
3. is is not obstructed by an obstacle

```
admissible(X,Wave,LastWave,Obst) :-
   adjacent(X,Wave,Obst),
   not member(X,LastWave),
   not member(X,Wave).

adjacent(X,Wave,Obstacles) :-
   member(X1,Wave),
   neighbor(X1,X),
   not obstructed(X,Obstacles).
```

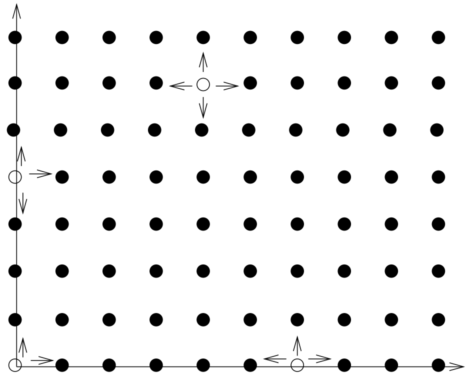

Lee's Algorithm…

- `next_to` takes a number `A` and returns `B=A+1` and `B=A-1`. `A-1` is returned only if the result is `>0`.
- `neighbor` uses `next_to` to generate neighboring points. The rules of `neighbor` state:
  1. The point `X2-Y` is a neighbor of point `X1-Y` if `X2 is X1+1`, or `X2=X1-1`. In other words, the first `neighbor` rule generates the points immediately above and below a given point.
  2. The point `X-Y2` is a neighbor of point `X-Y1` if `Y2 is Y1+1`, or `Y2=Y1-1`. In other words, the second `neighbor` rule generates the points immediately to the left and right of a given point.

Lee's Algorithm. . .

```
neighbor(X1-Y,X2-Y):- next_to(X1,X2).
neighbor(X-Y1,X-Y2):- next_to(Y1,Y2).

next_to(A,B) :- B is A+1.
next_to(A,B) :- A > 0, B is A-1.
```



Lee's Algorithm. . .

- `obstructed(Point,Obstacles)` checks to see if the point is on the perimeter of any of the obstacles in the list of obstacles `Obstacles`.
- The rule `obstructs(Point, Obstacle)` checks to see if the point is on the `perimeter` of the obstacle.

Note that `obstructed` is another generate-and-test procedure. `member` generates one obstacle at a time from this list, and `obstructs` checks to see if that obstacle obstructs the point.

Lee's Algorithm. . .

- `obstructed(Point,Obstacles)` checks to see if the point is on the perimeter of any of the obstacles in the list of obstacles `Obstacles`.
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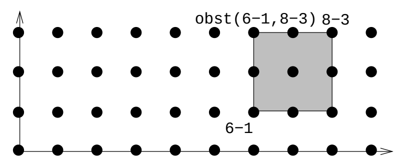
Note that `obstructed` is another generate-and-test procedure. `member` generates one obstacle at a time from this list, and `obstructs` checks to see if that obstacle obstructs the point.

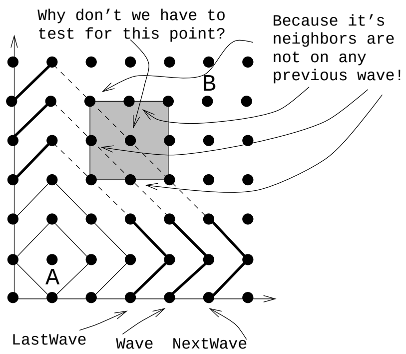
Lee's Algorithm. . .

```
% Generate an obstacle, then test
% if it obstructs a point Pt.
obstructed(Pt,Obsts) :-
   member(Obst,Obsts), obstructs(Pt,Obst).
obstructs(X-Y,obst(X-Y1,X2-Y2)) :-
   Y1=<Y, Y=<Y2.  % X-Y on bottom edge.
obstructs(X-Y,obst(X1-Y1,X-Y2)) :- Y1=<Y,Y=<Y2.
obstructs(X-Y,obst(X1-Y,X2-Y2)) :- X1=<X,X=<X2.
obstructs(X-Y,obst(X1-Y1,X2-Y)) :- X1=<X,X=<X2.
```



Lee's Algorithm…

- Why do we only need to check the perimeter? Shouldn't we have to check if a point lies *inside* an object as well?
- No, such points will never be considered. Their neighbors (which are on a perimeter) cannot be on a previous wave:



Lee's Algorithm…

The last part of the algorithm is to construct the actual path from the list of waves. The procedure `path` does this for us.

1. `path` starts by looking in the last wave for a neighbor of the destination node. In our example, the destination node is `5-5`, and a neighbor of `5-5` in the last wave is the node `5-4`.
2. `path` next looks for a neighbor for the new node in the next wave. Our example yields node `5-3` which is a neighbor of node `5-4`.
3. Eventually we'll get to the last wave which only contains the source node, in our case node `1-1`.

Lee's Algorithm…

```
aves = [[0-7,1-8,2-7,3-6,5-4,6-3,7-0,7-2,8-1],
        [0-6,1-7,2-6,5-3,6-0,6-2,7-1],
        [0-5,1-6,5-0,5-2,6-1],
        [0-4,1-5,4-0,4-2,5-1],
        [0-3,1-4,3-0,3-2,4-1],
        [0-0,0-2,1-3,2-0,2-2,3-1],
        [0-1,1-0,1-2,2-1],
        [1-1]]
ath(A,A,Waves,[A]) :- !.
ath(A,B,[Wave|Waves],[B|Path]) :-
  member(B1,Wave),
  neighbor(B,B1), !,
  path(A,B1,Waves,Path).
```


Readings and References

- Read Clocksin & Mellish, pp. 156--158.

Homework

Write Prolog predicates that given a database of countries and cities

```
% country(name, population, capital).
country(sweden, 8823, stockholm).
country(usa, 221000, washington).
country(france, 56000, paris).
% city(name, in_country, population).
city(lund, sweden, 88).
city(paris, usa, 1).  % Paris, Texas.
```

answer the following queries:

1. Which countries have cities with the same name as capitals of other countries?
2. In how many countries do more than $\frac{1}{3}$ of the population live in the capital?
3. Which capitals have a population more than 3 times larger than that of the secondmost populous city?