

CSc 372

Comparative Programming Languages

22 : Prolog — Lists

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Introduction

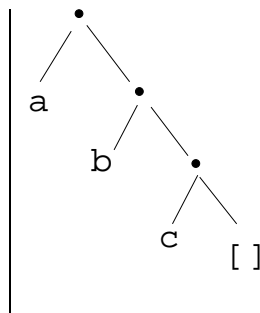
Prolog Lists

Haskell:

```
> 1 : 2 : 3 : []  
[1,2,3]
```

Prolog:

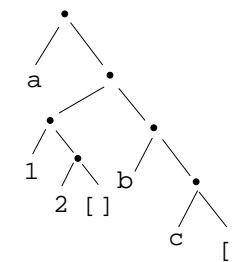
```
?- L = .(a, .(b, .(c, [])))  
L = [a, b, c]
```



- Both Haskell and Prolog build up lists using cons-cells.
- In Haskell the cons-operator is `:`, in Prolog `..`

Prolog Lists...

```
?- L = .(a, .(. (1, .(2, [])), .(b, .(c, []))))  
L = [a, [1, 2], b, c]
```



- Unlike Haskell, Prolog lists can contain elements of arbitrary type.

Matching Lists – [Head | Tail]

| A | F | $A \equiv F$ | variable subst. |
|---------------------------|----------------------|--------------|-----------------|
| <code>[]</code> | <code>[]</code> | yes | |
| <code>[]</code> | <code>a</code> | no | |
| <code>[a]</code> | <code>[]</code> | no | |
| <code> [[]]</code> | <code>[]</code> | no | |
| <code>[a [b, c]]</code> | <code>L</code> | yes | $L=[a,b,c]$ |
| <code>[a]</code> | <code>[H T]</code> | yes | $H=a, T=[]$ |

Matching Lists – [Head | Tail]...

| A | F | $A \equiv F$ | variable subst. |
|--------------------------|------------------------|--------------|-------------------|
| <code>[a, b, c]</code> | <code>[H T]</code> | yes | $H=a, T=[b,c]$ |
| <code>[a, [1, 2]]</code> | <code>[H T]</code> | yes | $H=a, T=[[1, 2]]$ |
| <code>[[1, 2], a]</code> | <code>[H T]</code> | yes | $H=[1,2], T=[a]$ |
| <code>[a, b, c]</code> | <code>[X, Y, c]</code> | yes | $X=a, Y=c$ |
| <code>[a, Y, c]</code> | <code>[X, b, Z]</code> | yes | $X=a, Y=b, Z=c$ |
| <code>[a, b]</code> | <code>[X, c]</code> | no | |

Prolog Lists — Member

Member

- ```
(1) member1(X, [Y|_]) :- X = Y.
(2) member1(X, [_|Y]) :- member1(X, Y).

(1) member2(X, [X|_]).
(2) member2(X, [_|Y]) :- member2(X, Y).

(1) member3(X, [Y|Z]) :- X = Y; member3(X, Z).
```

## Prolog Lists — Member...

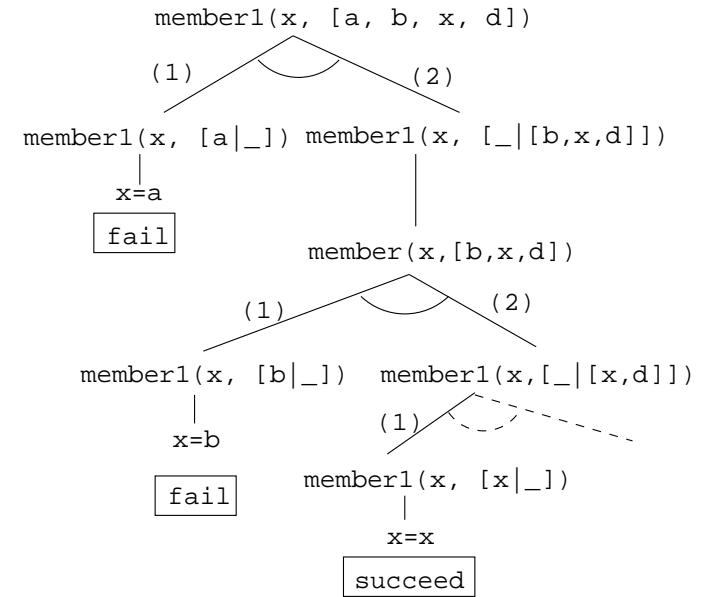
?- member(x, [a, b, c, x, f]).  
yes

?- member(x, [a, b, c, f]).  
no

?- member(x, [a, [x, y], f]).  
no

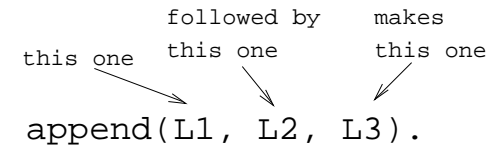
?- member(Z, [a, [x, y], f]).  
Z = a  
Z = [x, y]  
Z = f

## Prolog Lists — Member...



## Prolog Lists — Append

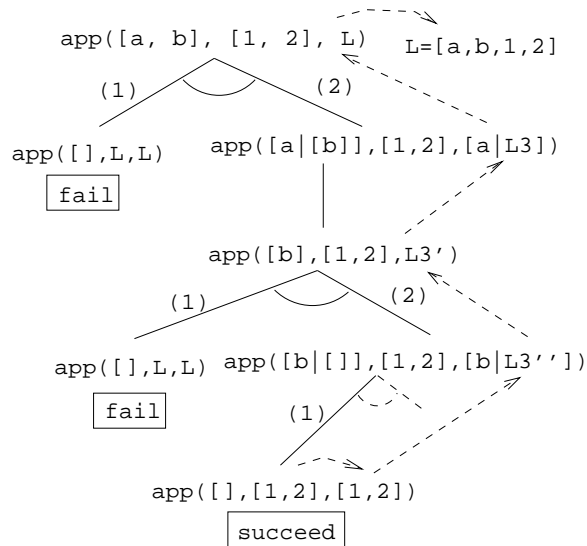
# Append



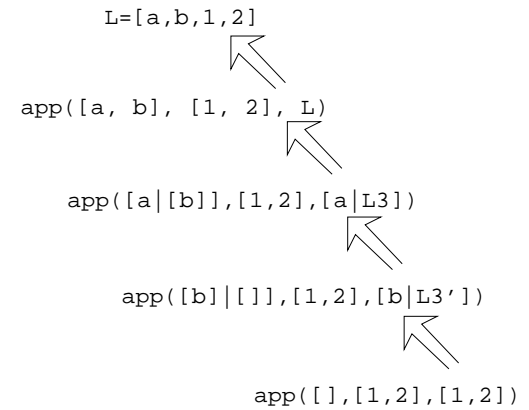
(1) append([], L, L)  
(2) append([X|L1], L2, [X|L3]) :-  
    append(L1, L2, L3).

- ① Appending  $L$  onto an empty list, makes  $L$ .
- ② To append  $L_2$  onto  $L_1$  to make  $L_3$ 
  - ① Let the first element of  $L_1$  be the first element of  $L_3$ .
  - ② Append  $L_2$  onto the rest of  $L_1$  to make the rest of  $L_3$ .

## Prolog Lists — Append...



## Prolog Lists — Append...



?- L = [a | L3], L3 = [b | L3'], L3' = [1,2].  
 L = [a,b,1,2], L3 = [b,1,2], L3' = [1,2]

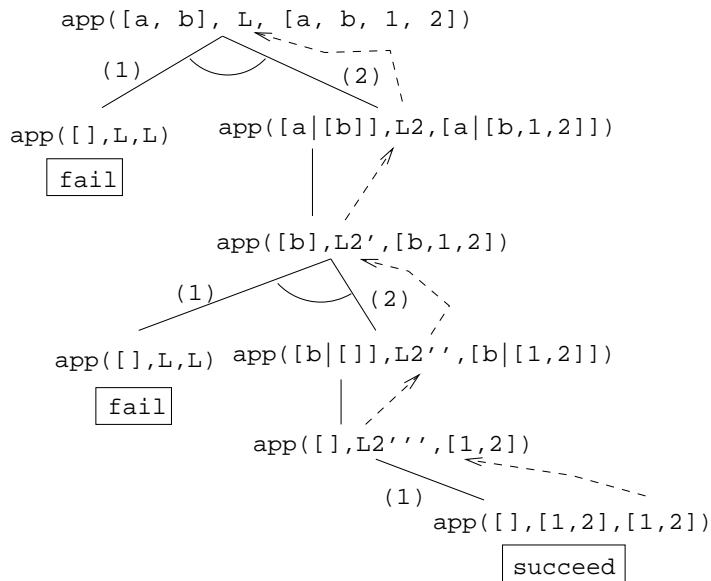
## Prolog Lists — Using Append

- 1 `append([a,b], [1,2], L)`
  - What's the result of appending [1,2] onto [a,b]?
- 2 `append([a,b], [1,2], [a,b,1,2])`
  - Is [a,b,1,2] the result of appending [1,2] onto [a,b]?
- 3 `append([a,b], L, [a,b,1,2])`
  - What do we need to append onto [a,b] to make [a,b,1,2]?
  - What's the result of removing the prefix [a,b] from [a,b,1,2]?

## Prolog Lists — Using Append...

- 4 `append(L, [1,2], [a,b,1,2])`
  - What do we need to append [1,2] onto to make [a,b,1,2]?
  - What's the result of removing the suffix [1,2] from [a,b,1,2]?
- 5 `append(L1, L2, [a,b,1,2])`
  - How can the list [a,b,1,2] be split into two lists L1 & L2?

## Prolog Lists — Using Append...



## Prolog Lists — Using Append...

?- append(L1, L2, [a,b,c]).

L1 = []

L2 = [a,b,c] ;

L1 = [a]

L2 = [b,c] ;

L1 = [a,b]

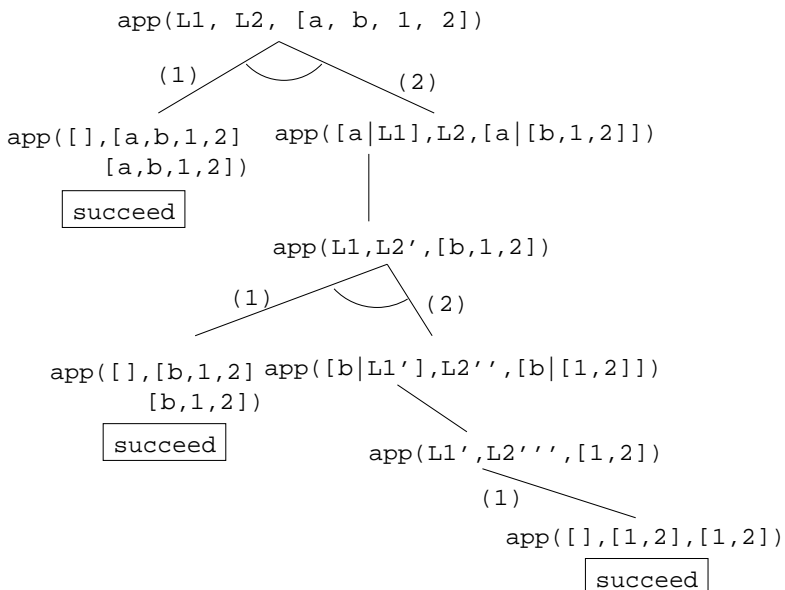
L2 = [c] ;

L1 = [a,b,c]

L2 = [] ;

no

## Prolog Lists — Using Append...



## Prolog Lists — Reusing Append

**member** Can we split the list Y into two lists such that X is at the head of the second list?

**adjacent** Can we split the list Z into two lists such that the two element X and Y are at the head of the second list?

**last** Can we split the list Y into two lists such that the first list contains all the elements except the last one, and X is the sole member of the second list?

```
member(X, Y) :- append(_, [X|Z], Y).
?- member(x, [a,b,x,d]).
```

```
adjacent(X, Y, Z) :- append(_, [X,Y|Q], Z).
?- adjacent(x,y,[a,b,x,y,d]).
```

```
last(X, Y) :- append(_, [X], Y).
?- last(x, [a,b,x]).
```

## Reversing a List

- `reverse1` is known as *naive reverse*.
- `reverse1` is *quadratic* in the number of elements in the list.
- From *The Art of Prolog*, Sterling & Shapiro pp. 12-13, 203.
- Is the basis for computing LIPS (Logical Inferences Per Second), the performance measure for logic computers and programming languages. Reversing a 30 element list (using naive reverse) requires 496 reductions. A reduction is the basic computational step in logic programming.

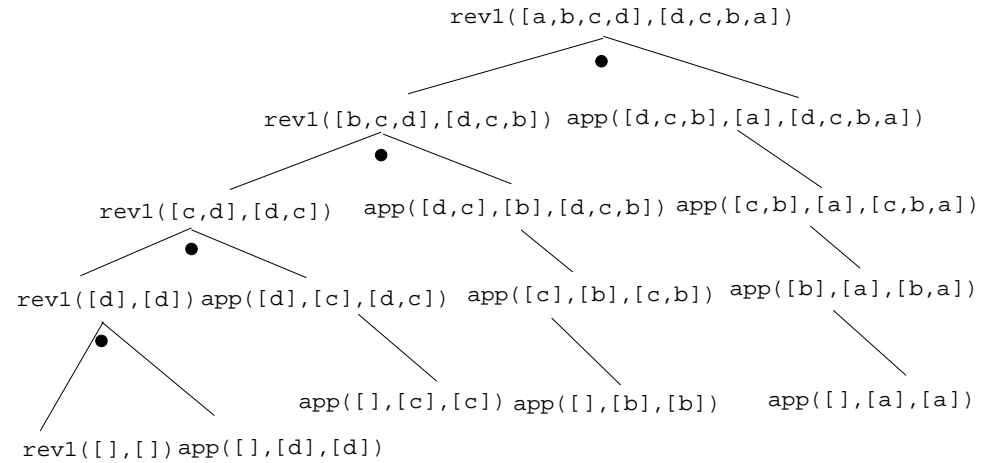
- `reverse1` works like this:
  - 1 Reverse the tail of the list.
  - 2 Append the head of the list to the reversed tail.
- `reverse2` is *linear* in the number of elements in the list.
- `reverse2` works like this:
  - 1 Use an accumulator pair `In` and `Out`
  - 2 `In` is initialized to the empty list.
  - 3 At each step we take one element (`X`) from the original list (`Z`) and add it to the beginning of the `In` list.
  - 4 When the original list (`Z`) is empty we instantiate the `Out` list to the result (the `In` list), and return this result up through the levels of recursion.

## Prolog Lists — Reverse...

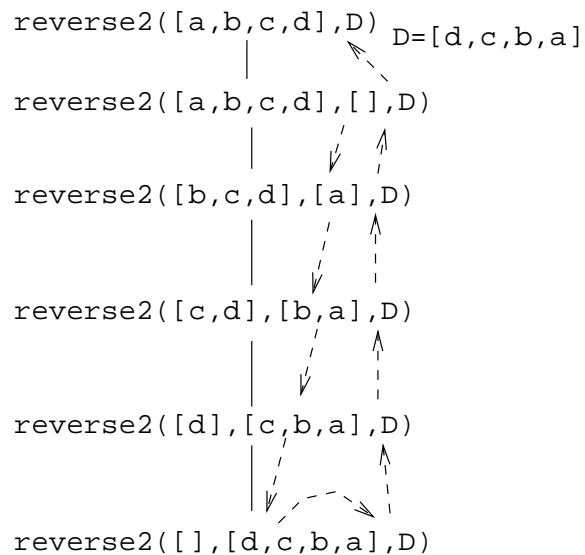
```
reverse1([], []).
reverse1([X|Q], Z) :-
 reverse1(Q, Y), append(Y, [X], Z).

reverse2(X, Y) :- reverse2(X, [], Y).
reverse2([X|Z], In, Out) :-
 reverse(Z, [X|In], Out).
reverse2([], Y, Y).
```

## Reverse – Naive Reverse



## Reverse – Smart Reverse



Delete

delete      from this      to yield  
 this one   list      this list  
 ↘                      ↙  
 delete(X, L1, L2).

- delete\_one      ● Remove the first occurrence.
- delete\_all      ● Remove all occurrences.
- delete\_struct      ● Remove all occurrences from all levels of a list of lists.

```
?- delete_one(x, [a, x, b, x], D).
 D = [a, b, x]
?- delete_all(x, [a, x, b, x], D).
 D = [a, b]
?- delete_all(x, [a, x, b, [c, x], x], D).
 D = [a, b, [c, x]]
?- delete_struct(x, [a, x, [c, x], v(x)], D).
 D = [a, b, [c], v(x)]
```

## delete\_one

- ❶ If X is the first element in the list then return the tail of the list.
- ❷ Otherwise, look in the tail of the list for the first occurrence of X.

## delete\_all

- ❶ If the head of the list is X then remove it, and remove X from the tail of the list.
- ❷ If X is *not* the head of the list then remove X from the tail of the list, and add the head to the resulting tail.
- ❸ When we're trying to remove X from the empty list, just return the empty list.



- Why do we test for the recursive boundary case (`delete_all(X, [], [])`) last? Well, it only happens once so we should perform the test as few times as possible.
- The reason that it works is that when the original list (the second argument) is `[],` the first two rules of `delete_all` won't trigger. Why? Because, `[]` does not match `[H|T],` that's why!

`delete_struct`

- 1 The first rule is the same as the first rule in `delete_all.`
- 2 The second rule is also similar, only that we descend into the head of the list (in case it should be a list), as well as the tail.
- 3 The third rule is the catch-all for lists.
- 4 The last rule is the catch-all for non-lists. It states that all objects which are not lists (atoms, integers, structures) should remain unchanged.

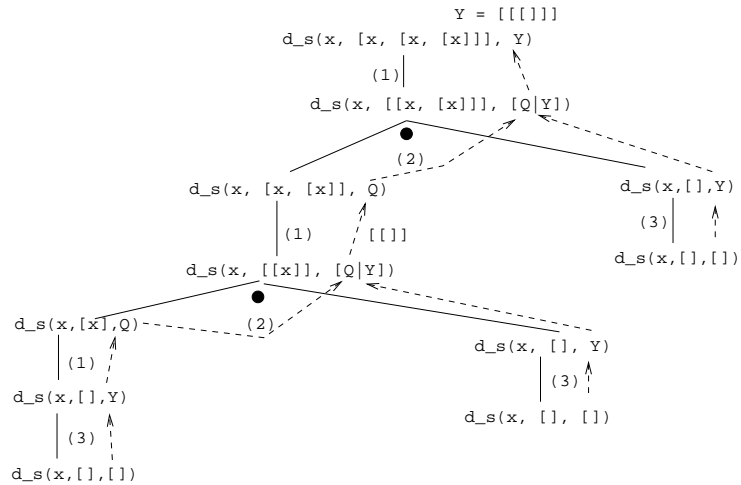
```
delete_one(X, [X|Z], Z).
delete_one(X, [V|Z], [V|Y]) :-
 X \== V,
 delete_one(X, Z, Y).
```

```
delete_all(X, [X|Z], Y) :- delete_all(X, Z, Y).
delete_all(X, [V|Z], [V|Y]) :-
 X \== V,
 delete_all(X, Z, Y).
delete_all(X, [], []).
```

```
(1) delete_struct(X, [X|Z], Y) :-
 delete_struct(X, Z, Y).

(2) delete_struct(X, [V|Z], [Q|Y]) :-
 X \== V,
 delete_struct(X, V, Q),
 delete_struct(X, Z, Y).

(3) delete_struct(X, [], []).
(4) delete_struct(X, Y, Y).
```



## Application: Sorting

### Sorting – Naive Sort

```
permutation(X, [Z|V]) :-
 delete_one(Z, X, Y),
 permutation(Y, V).
permutation([], []).
```

```
ordered([X]).
ordered([X, Y|Z]) :-
 X =< Y,
 ordered([Y|Z]).
```

```
naive_sort(X, Y) :-
 permutation(X, Y),
 ordered(Y).
```

### Sorting – Naive Sort...

- This is an application of a Prolog cliché known as **generate-and-test**.

#### naive\_sort

- The permutation part of naive\_sort generates one possible permutation of the input
- The ordered predicate checks to see if this permutation is actually sorted.
- If the list still isn't sorted, Prolog backtracks to the permutation goal to generate an new permutation, which is then checked by ordered, and so on.

## Sorting – Naive Sort...

### permutation

- ① If the list is not empty we:
  - ① Delete some element Z from the list
  - ② Permute the remaining elements
  - ③ Add Z to the beginning of the list

When we backtrack (ask `permutation` to generate a new permutation of the input list), `delete_one` will delete a different element from the list, and we will get a new permutation.

- ② The permutation of an empty list is the empty list.
- Notice that, for efficiency reasons, the boundary case is put *after* the general case.

## Sorting – Naive Sort...

`delete_one` Removes the first occurrence of X (its first argument) from V (its second argument).

- Notice that when `delete_one` is called, its first argument (the element to be deleted), is an uninstantiated variable. So, rather than deleting a specific element, it will produce the elements from the input list (+ the remaining list of elements), one by one:

```
?- delete_one(X, [1,2,3,4], Y).
```

```
X = 1, Y = [2,3,4] ;
X = 2, Y = [1,3,4] ;
X = 3, Y = [1,2,4] ;
X = 4, Y = [1,2,3] ;
no.
```

## Sorting – Naive Sort...

The proof tree in the next slide illustrates `permutation([1,2,3],V)`. The dashed boxes give variable values for each backtracking instance:

**First instance:** `delete_one` will select  $X=1$  and  $Y=[2,3]$ .  $Y$  will then be permuted into  $Y'=[2,3]$  and then (after having backtracked one step)  $Y'=[3,2]$ . In other words, we generate  $[1,2,3]$ ,  $[1,3,2]$ .

**Second instance:** We backtrack all the way back up the tree and select  $X=2$  and  $Y=[1,3]$ .  $Y$  will then be permuted into  $Y'=[1,3]$  and then  $Y'=[3,2]$ . In other words, we generate  $[2,1,3]$ ,  $[2,3,1]$ .

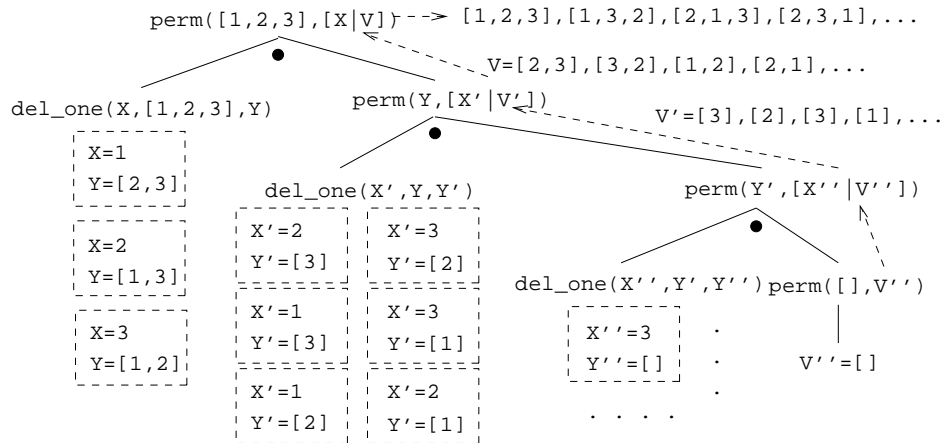
## Sorting – Naive Sort...

**Third instance:** Again, we backtrack all the way back up the tree and select  $X=3$  and  $Y=[1,2]$ . We generate  $[3,1,2]$ ,  $[3,2,1]$ .

```
?- permutation([1,2,3],V).
```

```
V = [1,2,3] ;
V = [1,3,2] ;
V = [2,1,3] ;
V = [2,3,1] ;
V = [3,1,2] ;
V = [3,2,1] ;
no.
```

## Permutations



## Sorting Strings

- Prolog strings are lists of ASCII codes.
- "Maggie" = [77,97,103,103,105,101]

```

aless(X,Y) :-
 name(X,X1), name(Y,Y1),
 alessx(X1,Y1).

```

```

alessx([],[_|_]).
alessx([X|_],[Y|_]) :- X < Y.
alessx([A|X],[A|Y]) :- alessx(X,Y).

```

## Mutant Animals

## Application: Mutant Animals

- From *Prolog by Example*, Coelho & Cotta.
- We're given a set of words (French animals, in our case).
- Find pairs of words where the ending of the first one is the same as the beginning of the second.
- Combine the words, so as to form new "mutations".

## Mutant Animals...

- 1 Find two words, Y and Z.
- 2 Split the words into lists of characters. `name(atom, list)` does this.
- 3 Split Y into two sublists, Y1 and Y2.
- 4 See if Z can be split into two sublists, such that the prefix is the same as the suffix of Y (Y2).
- 5 If all went well, combine the prefix of Y (Y1) with the suffix of Z (Z2), to create the mutant list X.
- 6 Use `name` to combine the string of characters into a new atom.

## Mutant Animals...

```
mutate(M) :-
 animal(Y), animal(Z), Y \== Z,
 name(Y,Ny), name(Z,Nz),
 append(Y1,Y2,Ny), Y1 \== [],
 append(Y2, Z2, Nz), Y2 \== [],
 append(Y1,Nz,X), name(M,X).

animal(alligator). /* crocodile*/
animal(tortue). /* turtle */
animal(caribou). /* caribou */
animal(ours). /* bear */
animal(cheval). /* horse */
animal(vache). /* cow */
animal(lapin). /* rabbit */
```

## Mutant Animals...

```
?- mutate(X).
X = alligatortue ; /* alligator+ tortue */
X = caribours ; /* caribou + ours */
X = chevalligator ; /* cheval + alligator*/
X = chevalapin ; /* cheval + lapin */
X = vacheval /* vache + cheval */
```

## Summary

## Prolog So Far...

- Lists are nested *structures*
- Each list node is an object
  - with functor `.` (dot).
  - whose first argument is the head of the list
  - whose second argument is the tail of the list
- Lists can be split into head and tail using `[H|T]`.
- Prolog strings are lists of ASCII codes.
- `name(X,L)` splits the atom `X` into the string `L` (or vice versa).