

- We have already used a number of polymorphic functions that are defined in the standard prelude.

- `head` is a function from "lists-of-things" to "things":

```
head :: [a] -> a
```

- `tail` is a function from lists of elements of some type, to a list of elements of the same type:

```
tail :: [a] -> [a]
```

- `cons` "(::)" takes two arguments: an element of some type `a` and a list of elements of the same type. It returns a list of elements of type `a`:

```
(:) :: a -> [a] -> [a]
```

- In many languages we can't write a **generic** sort routine, i.e. one that can sort arrays of integers as well as arrays of reals:

```
procedure Sort (
  var A : array of <type>;
  n : integer);
```

- In Haskell (and many other FP languages) we can write **polymorphic** ("many shapes") functions.

- Functions of polymorphic type are defined by using **type variables** in the signature:

```
length :: [a] -> Int
length s = ...
```

- `length` is a function from lists of elements of some (unspecified) type `a`, to integer. I.e. it doesn't matter if we're taking the length of a list of integers or a list of reals or strings, the algorithm is the same.

```
length [1,2,3] => 3 (list of Int)
```

```
length ["Hi", "there", "!!"] => 3 (list of String)
length "Hi!" => 3 (list of Char)
```

- Obviously `remdups` should work for any list, not just lists of `Ints`. Removing duplicates from a list of strings is no different from removing duplicates from a list of integers.
- However, there's a complication. In order to remove duplicates from a list, we must be able to compare list elements for equality.
- The polymorphic type `[a] -> [a]` is therefore a bit too general, since it would allow any type, even one for which equality is not defined.

Context Predicates

- Remember the `remdups` function:


```
remdups [1]      => [1]
remdups [1,2,1] => [1,2,1]
remdups [1,2,1,1,2] => [1,2,2]
remdups [1,1,1,1,2] => [1,2,1]
```
- Algorithm in Haskell:


```
remdups :: [Int] -> [Int]
remdups x:y:xs =
  if x == y then
    remdups y:xs
  else
    x : remdups y:xs
remdups xs = xs
remdups xs => case 3
remdups xs => case 2
remdups xs => case 1
```

Polymorphic Functions...

- Note that `head` and `tail` always take a list as their argument. `tail` always returns a list, but `head` can return any type of object, including a list.
 - Note that it is because of Haskell's strong typing that we can only create lists of the same type of element. If we tried to do `[True] 5 :` ?
- the Haskell type checker would complain that we were consing an `Int` onto a list of `Bools`, while the type of `":"` is `(:) :: a -> [a] -> [a]`

- We want to define functions that are as **reusable** as possible.
 - 1 **Polymorphic** functions are reusable because they can be applied to arguments of different types.
 - 2 **Curried** functions are reusable because they can be **specialized**; i.e. from a curried function f we can create a new function f' simply by "plugging in" values for some of the arguments, and leaving others undefined.

Multiple Context Predicates

- Consider the `signum` function:


```
signum :: (Num a, Ord a) => a -> Int
signum n | n == 0 = 0
         | n < 0 = -1
         | n > 0 = 1
```
- `signum` can be applied to any type that is a number (hence the `Num` a predicate), and for which the relational operators are defined (`Ord a`).
- Without these restrictions, the polymorphic `signum` function could have been applied to lists, for example, which would not have made sense.

Context Predicates...

- Haskell uses **context predicates** to restrict polymorphic types:


```
remdups :: Eq [a] => [a] -> [a]
```

Now, `remdups` may only be applied to list of elements where the element type has `==` and `\=` defined.
- `Eq` is called a **type class**. `Ord` is another useful type class. It is used to restrict the polymorphic type of a function to types for which the relational operators (`<`, `<=`, `>`, `>=`) have been defined.

Conclusion

Homework

- Let f be a function from Int to Int , i.e. $f :: \text{Int} \rightarrow \text{Int}$. Define a function $\text{total } f$ x so that $\text{total } f$ is the function which at value n gives the total $f\ 0 + f\ 1 + \dots + f\ n$.

Example:

```
double x = 2*x
pow2 x = x^2
total double = total pow2
total pow2 = total pow
total double 5
30
total pow 5
55
```

Homework

- Define a polymorphic function $\text{copy } n$ x which returns a list of n copies of x .

Example:

```
? copy 5 "five"
["five", "five", "five", "five", "five"]
? copy 5 5
[5, 5, 5, 5, 5]
? copy 5 (dup 5)
[(5, 5), (5, 5), (5, 5), (5, 5), (5, 5)]
```

Homework

- Define a polymorphic function $\text{dup } x$ which returns a tuple with the argument duplicated.

Example:

```
? dup 1
(1, 1)
? dup "Hello, we again!"
("Hello, we again!", "Hello, we again!")
? dup (dup 3.14)
(3.14, 3.14), (3.14, 3.14)
```

Homework

- A polymorphic function is defined using **type variables** in the signature. A type variable can represent an **arbitrary** type.
- All occurrences of a particular type variable appearing in a type signature must represent the same type.
- An identifier will be treated as an operator symbol if it is enclosed in backquotes: "```".
- An operator symbol can be treated as an identifier by enclosing it in parenthesis: `(+)`.