CSc 372

Comparative Programming Languages

6 : Haskell — Lists

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The List Datatype

- All functional programming languages have the ConsList ADT built-in. It is called so because lists are constructed by "consing" (adding) an element on to the beginning of the list.
- Lists are defined recursively:
  1. The empty list [ ] is a list.
  2. An element x followed by a list L (x:L), is a list.
- Examples:

```
[ ]
2:[ ]
3:(2:[ ])
4:(3:(2:[ ]))
```

The List Datatype. . .

- The cons operator ":" is right associative (it binds to the right, i.e.

  $$\texttt{1:2:[ ]} \equiv \texttt{1:(2:[ ])}$$

  so

  $$\texttt{3:(2:[ ])}$$

  can be written without brackets as

  $$\texttt{3:2:[ ]}$$

The List Datatype...

- Lists can also be written in a convenient bracket notation.

  `2:[ ]` ⇒ `[2]`
  `3:(2:[ ])` ⇒ `[3,2]`
  `4:(3:(2:[ ])` ⇒ `[4,3,2]`

- You can make lists-of-lists (`[[1],[5]]`), lists-of-lists-of-lists (`[[[1,2]],[[3]]]`), etc.

The List Datatype. . .

- More cons examples:

```
1:[2,3]            ⇒ [1,2,3]
[1]:[[2],[3]]      ⇒ [[1],[2],[3]]
```

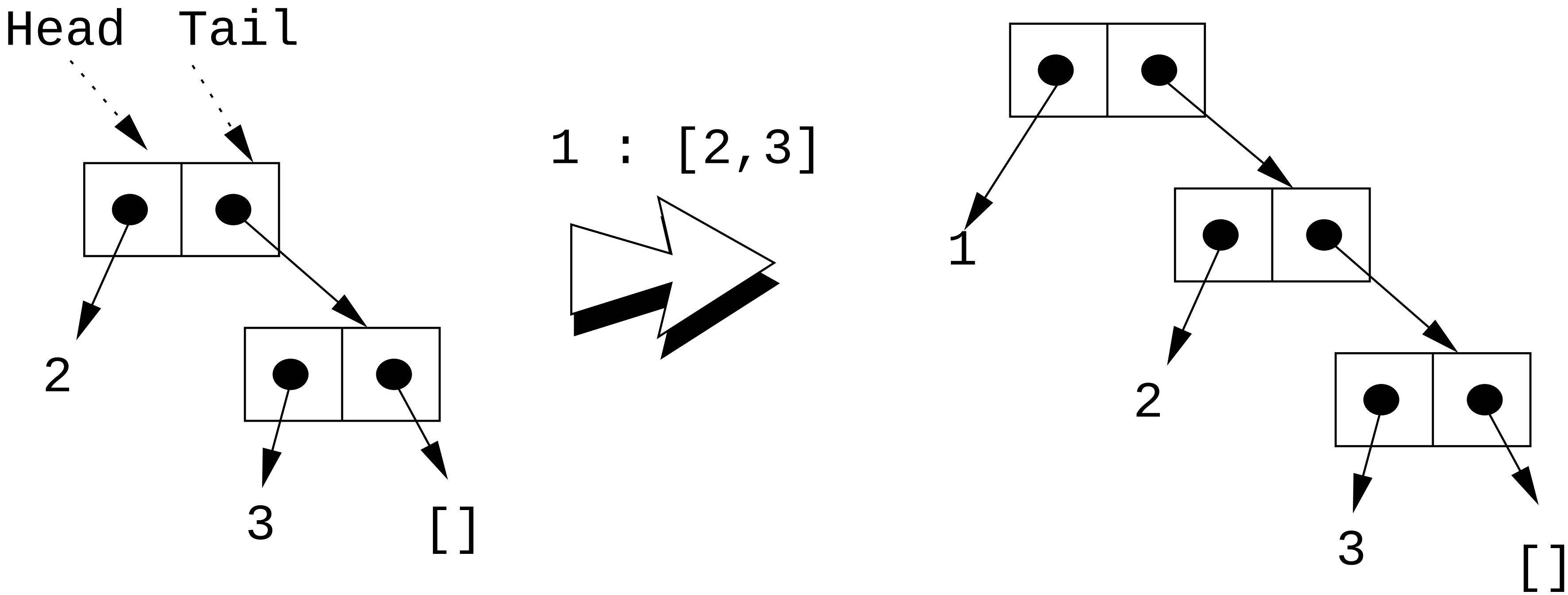
- Note that the elements of a list must be of the same type!

```
[1,[1],1]          ⇒ Illegal!
[[1],[2],[[3]]]    ⇒ Illegal!
[1,True]           ⇒ Illegal!
```

Internal Representation

- Internally, Haskell lists are represented as linked cons-cells.
- A cons-cell is like a C `struct` with two pointer fields `head` and `tail`.
- The `head` field points to the first element of the list, the `tail` field to the rest of the list.
- The `:`-operator creates a new cons-cell (using `malloc`) and fills in the `head` and `tail` fields to point to the first element of the new list, and the rest of the list, respectively.

Internal Representation — Example



`2:3:[]` or `[2,3]` `1:2:3:[]` or `[1,2,3]`

Standard Operations on Lists

`head` and `tail`

- The Standard Prelude has many built-in operations on lists.
- Two principal operators are used to take lists apart:
  1. `head L` – returns the first element of L.
  2. `tail L` – returns L without the first element.
- The cons operator `":"` is closely related to `head` and `tail`:
  1. `head (x:xs)` ≡ `x`
  2. `tail (x:xs)` ≡ `xs`
- The cons operator `":"` constructs new lists, `head` and `tail` take them apart.

head and tail...

```
head [1,2,3]     ⇒ 1
tail [1,2,3]     ⇒ [2,3]
tail [1]         ⇒ [ ]    ([1] == 1:[ ])
head [ ]         ⇒ ERROR
tail [ ]         ⇒ ERROR
head (1:[2,3])   ⇒ 1
tail (1:[2,3])   ⇒ [2,3]
head (tail [1,2,3]) ⇒ 2
head (tail [[1],[2],[3]]) ⇒ [2]
```

length and ++

- `length xs` – Number of elements in the list `xs`.
- `xs ++ ys` – The elements of `xs` followed by the elements of `ys`.

Examples:

```
length [1,2,3]           ⇒ 3
length [ ]               ⇒ 0
[1,2] ++ [3,4]           ⇒ [1,2,3,4]
[1,2] ++ [ ]             ⇒ [1,2]
[1] ++ [2,3] ++ [4]      ⇒ [1,2,3,4]
length ([1]++[2,3])      ⇒ 3
[1] ++ [length [2,3]]    ⇒ [1,2]
```

`concat`

- `concat xss` – all of the lists in `xss` appended together.

`concat [[1],[4,5],[6]]` $\Rightarrow$ `[1,4,5,6]`

- Note that `concat` takes a list of lists as argument.

map

- `map f xs` – list of values obtained by applying the function `f` to the values in `xs`.

```
map even [1,2,3]  ⇒ [False,True,False]
map square [1,2,3]      ⇒ [1,4,9]
```

- Note that `map` takes a function as its first argument. A function which takes a function as an argument or delivers one as its result, is called a higher-order function.
- We will talk more about higher-order functions in future lectures.

More list operation examples

```
head ([1,2] ++ [3,4]) ⇒
   head [1,2,3,4] ⇒ 1
tail (concat [[1],[3,4],[5]]) ⇒
   tail [1,3,4,5] ⇒ [3,4,5]
tail (map double (concat [[1],[3],[4]])) ⇒
   tail (map double [1,3,4]) ⇒
   tail [2,6,8] ⇒ [6,8]
```

The String Type

- A Haskell string is a list of characters:

  `type String = [Char]`
- All list manipulation functions can be applied to strings.
- Note that `"" == []`.

```
"Chris"        ⇔ ['C','h','r','i','s']
head "Chris"   ⇔ 'C'
tail "Chris"   ⇔ ['h','r','i','s']
"Chris" ++ "tian" ⇔
   ['C','h','r','i','s','t','i','a','n']
map ord "Hello" ⇔
   [72,101,108,108,111]
concat ["Have ","a ","cow, ","man!"]
   ⇔ "Have a cow, man!"
```

Recursion Over Lists

Recursion on the Tail

- Compute the length of a list.
- This is called recursion on the tail.

```
len ::  [Int] -> Int
len xs = if xs == [] then
         0
      else
         1 + len (tail xs)
```

Variable Naming Conventions

- When we write functions over lists it's convenient to use a consistent variable naming convention. We let
  - `x`, `y`, `z`, $\cdots$ denote list elements.
  - `xs`, `ys`, `zs`, $\cdots$ denote lists of elements.
  - `xss`, `yss`, `zss`, $\cdots$ denote lists of lists of elements.

Map Function

- Map a list of numbers to a new list of their absolute values.
- In the previous examples we returned an `Int` — here we're mapping a list to a new list.
- This is called a map function.

```
abslist ::  [Int] -> [Int]
abslist xs = if xs == [] then
          []
      else
         abs (head xs) :  abslist (tail xs)
```

Map Function...

```
> abslist []
[]
> abslist [1]
[1]
 abslist [1,-2]
[1,2]
```

Recursion Over Two Lists

- `listeq xs ys` returns `True` if two lists are equal.

```
listeq ::  [Int] -> [Int] -> Bool
listeq xs ys = if xs==[] && ys==[] then
                  True
               else if xs==[] || ys==[] then
                  False
               else if head xs /= head ys then
                  False
               else
                  listeq (tail xs) (tail ys)
```

Recursion Over Two Lists...

```
> listeq [1] [2]
False
> listeq [1] [1]
True
> listeq [1] [1,2]
False
> listeq [1,2] [1,2]
True
```

Append

- `append xs ys` takes two lists as arguments and returns a new list, consisting of the elements of `xs` followed by the elements of `ys`.
- To do this recursively, we take `xs` apart on the way down into the recursion, and "attach" them to `ys` on the way up:

```
append ::  [Int] -> [Int] -> [Int]
append xs ys = if xs==[] then
         ys
      else
         (head xs) :  (append (tail xs) ys)
```

Append...

```
> append [] []
[]
> append [1] []
[1]
> append [1] [2]
[1,2]
> append [1,2,3] [4,5,6]
[1,2,3,4,5,6]
```

Arithmetic Sequences

Arithmetic Sequences

- Haskell provides a convenient notation for lists of numbers where the difference between consecutive numbers is constant.

```
[1..3]  ⇒  [1,2,3]
[5..1]  ⇒  []
```

- A similar notation is used when the difference between consecutive elements is $\neq 1$: Examples:

```
[1,3..9]     ⇒  [1,3,5,7,9]
[9,8..5]     ⇒  [9,8,7,6,5]
[9,8..11]    ⇒  []
```

  Or, in general:

```
[m,k..n]  ⇒
    [m,m+(k-m)*1,m+(k-m)*2,···,n]
```

Arithmetic Sequences...

- Or, in English

  "`m` and `k` are the first two elements of the sequence. All consecutive pairs of elements have the same difference as `m` and `k`. No element is greater than `n`."

- Or, in some other words,

  "`m` and `k` form a prototype for consecutive element pairs in the list."

- Later in the course we will talk about infinite lists. Haskell has the capability to create infinite arithmetic sequences:

  `[3..]` $\Rightarrow$ `[3,4,5,6,7,`$\cdots$`]`

  `[4,3..]` $\Rightarrow$ `[4,3,2,1,0,-1,-2,`$\cdots$`]`

Summary

- The bracketed list notation `[1,2,3]` is just an abbreviation for the list constructor notation `1:2:3:[]`.
- Lists can contain anything: integers, characters, tuples, other lists, but every list must contain elements of the same type only.
- `:`, `++`, `concat`, and list comprehensions create lists.
- `head` and `tail` take lists apart.

Summary...

- The notation `[m..n]` generates lists of integers from `m` to `n`.
- If the difference between consecutive integers is $\neq 1$, we use the slightly different notation `[m,k..n]`. The first two elements of the generated list are `m` and `k`. The remaining elements are as far apart as `m` and `k`.

Homework

- Which of the following are legal list constructions? First work out the answer in your head, then try it out with the `hugs` interpreter.

1. `1 : []`
2. `1 : [] : []`
3. `1 : [1]`
4. `[] : [1]`
5. `[1] : [1] : []`

Homework

- Show the lists generated by the following Haskell list expressions.

1. `[7..11]`
2. `[11..7]`
3. `[3,6..12]`
4. `[12,9..2]`

Homework

1. Write a function `getelmt xs n` which returns the *n*:th element of a list of integers.
2. Write a function `evenelmts xs` which returns a new list consisting of the 0:th, 2:nd, 4:th, . . . elements of an integer list `xs`.

Homework

- For each of the function signatures on the next slide, describe in words what type of function they represent. For example, for `f1` you'd say "this is a function which takes one Int argument and returns and Int result."
- Also, for each signature, give an example of a function that would have this signature. For example, "`f1` could be the `abs` function which takes an Int as argument and returns its absolute value."

Homework...

1. `f1 :: Int -> Int`
2. `f2 :: Int -> Bool`
3. `f3 :: (Int,Int)->Int`
4. `f4 :: [Int] -> Int`
5. `f5 :: [Int] -> Bool`
6. `f6 :: [Int]->Int->Bool`
7. `f7 :: [Int]->[Int]->[Int]`
8. `f8 :: [[Int]]->[Int]`
9. `f9 :: [Int]->[Int]`
10. `f10 :: [Int]->[Bool]`