CSc 453

# Compilers and Systems Software

## 6 : Top-Down Parsing I

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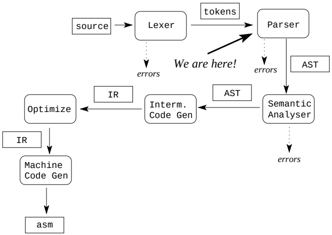
# Overview

## Compiler Phases




# Grammars

## Context Free Grammars

- CFGs are used to describe the syntax of programming languages. A production

$$S \rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$$

in a CFG says

*"If $S_1$ and $S_2$ are statements and $E$ an expression then '$\underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$' is a statement".*

Notice that this production is recursive; it allows if-statements to occur within if-statements.

## Context Free Grammars...

$$S \rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$$

- $\underline{\texttt{if}}$, $\underline{\texttt{then}}$, and $\underline{\texttt{else}}$ are terminal symbols or tokens.
- $S$, $S_1$, $S_2$, and $E$ are non-terminals. They are like "variables", that represent the kinds of strings that the grammar defines as statements or expressions, respectively.

## CFG Notation

terminals:

$\underline{a}, \underline{b}, \underline{c}, \cdots, \underline{+}, \underline{-}, \cdots, \underline{0}, \underline{1}, \cdots, \underline{\texttt{if}}, \underline{\texttt{do}}$.

nonterminals:

$A, B, C, \cdots, S, \cdots, \mathrm{expr}, \mathrm{stmt}$.

grammar symbols:

$X, Y, Z, \cdots$ (either terminals or nonterminals).

strings of terminals:

$u, v, w \cdots$.

strings of grammar symbols:

$\alpha, \beta, \gamma, \cdots$ (strings of terminals or nonterminals).

productions:

$A \rightarrow \alpha_1, A \rightarrow \alpha_2, \cdots, A \rightarrow \alpha_k$, or

$A \rightarrow \alpha_1 \mid \alpha_2 \cdots \mid \alpha_k$.

## Derivations — Productions as *Rewrite Rules*

1. Start with the start symbol, $S$.
2. Pick any production $S \rightarrow \alpha$, eg. $S \rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$.
3. We say that $S$ derives $\underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$, or $S \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$. '$\underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$' is a sentential form derived from $S$.
4. Repeat: pick a nonterminal $A$ from the sentential form, replace with the RHS of a production $A \rightarrow \alpha$:
   $S \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E\underline{+}E \Rightarrow$
   $\underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id}} + E \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id+num}}$. $S \stackrel{*}{\Rightarrow} \underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id+num}}$.

$$\begin{array}{lcl} S & \rightarrow & \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \mid \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\ E & \rightarrow & E\underline{+}E \mid \underline{\texttt{id}} \mid \underline{\texttt{num}} \end{array}$$

## Terminology

- A grammar is a 4-tuple

  (non-terminals, terminals, productions, start-symbol)

  or

  $$(N, \Sigma, P, S)$$

- A production is of the form $\alpha \to \beta$ where $\alpha, \beta$ are taken from $N \bigcup \Sigma$.
- Read $\alpha \to \beta$ as "rewrite $\alpha$ with $\beta$".
- Read $\Rightarrow$ as "directly derives".
- Read $\stackrel{r}{\Rightarrow}$ as "directly derives using rule $r$".
- Read $\stackrel{*}{\Rightarrow}$ as "derives in zero or more steps".

## Derivations...

- $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if
  - $\boxed{A \to \gamma}$ is a production, and
  - $\alpha$ and $\beta$ are strings of grammar symbols.

$\Rightarrow$: Derives in one step.

$\stackrel{*}{\Rightarrow}$: Derives in 0 or more steps.

$\stackrel{+}{\Rightarrow}$: Derives in 1 or more steps.

$\underset{lm}{\Rightarrow}$: Leftmost derivation.

$\underset{rm}{\Rightarrow}$: Rightmost derivation.

$L(G)$: The language generated by grammar $G$. This is the set of strings $w$, such that there is a derivation $S \stackrel{+}{\Rightarrow} w$, where $S$ is $G$'s start-symbol.

## Derivations...

The string of terminal symbols $\boxed{\underline{\texttt{id:=id+num}}}$ is generated by a leftmost derivation:

$$
\begin{aligned}
S &\underset{lm}{\Rightarrow} \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \underset{lm}{\Rightarrow} \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E\underline{+}E \\
&\underset{lm}{\Rightarrow} \underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id}}\underline{+}E \underset{lm}{\Rightarrow} \underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id+num}} \\
S &\stackrel{+}{\Rightarrow} \underline{\texttt{id:=id+num}}
\end{aligned}
$$

Example Grammar:

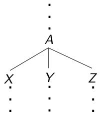
$$
\begin{aligned}
S &\to \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \mid \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\
E &\to E\underline{+}E \mid \underline{\texttt{id}} \mid \underline{\texttt{num}}
\end{aligned}
$$

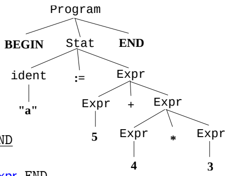
## Parse Trees...

- If one step of our derivation is

  $$\cdots\ A\ \cdots \Rightarrow \cdots\ X\ Y\ Z\ \cdots$$

  (i.e, we used the rule $A \to XYZ$) then we'll get a parse (sub-)tree





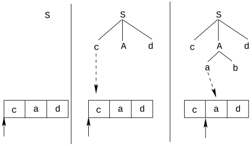

Program ⇒ BEGIN Stat END
⇒ BEGIN ident := Expr END
⇒ BEGIN "a" := Expr END
⇒ BEGIN "a" := Expr + Expr END
⇒ BEGIN "a" := 5 + Expr END
⇒ BEGIN "a" := 5 + Expr * Expr END
⇒ BEGIN "a" := 5 + 4 * Expr END
⇒ BEGIN "a" := 5 + 4 * 3 END

# Top-Down Parsing

## Top-Down Backtracking Parser

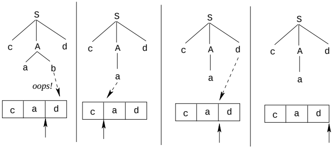
- Top-down parsing involves building a parse tree for the input string by starting at the root and adding nodes in preorder.

$$S \rightarrow cAd \qquad A \rightarrow ab|a$$




## Top-Down Backtracking Parser. . .

- If a backtracking top-down parser chooses the wrong production rule to expand a node it backs up over the input, and undoes some of the parse tree construction:



# Grammar Rewriting

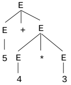
## Ambiguous Grammars

- A grammar is ambiguous if some string of tokens can produce two (or more) different parse trees.

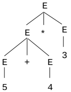
$$E ::= E \ \underline{+}\ E \mid E \ \underline{*}\ E \mid \underline{\texttt{number}}$$

$$\boxed{\texttt{5 + 4 * 3}}$$

$$\begin{aligned} E &\Rightarrow E \ \underline{+}\ E \\ &\Rightarrow 5 \ \underline{+}\ E \\ &\Rightarrow 5 \ \underline{+}\ E \ \underline{*}\ E \\ &\Rightarrow 5 \ \underline{+}\ 4 \ \underline{*}\ E \\ &\Rightarrow 5 \ \underline{+}\ 4 \ \underline{*}\ 3 \end{aligned}$$




$$\begin{aligned} E &\Rightarrow E \ \underline{*}\ E \\ &\Rightarrow E \ \underline{*}\ 3 \\ &\Rightarrow E \ \underline{+}\ E \ \underline{*}\ 3 \\ &\Rightarrow E \ \underline{+}\ 4 \ \underline{*}\ 3 \\ &\Rightarrow 5 \ \underline{+}\ 4 \ \underline{*}\ 3 \end{aligned}$$




## Operator Precedence

- The precedence of an operator is a measure of its binding power, i.e. how strongly it attracts its operands.
- Usually $*$ has higher precedence than $+$:

$$4 + 5 * 3$$

means

$$4 + (5 * 3),$$

not

$$(4 + 5) * 3.$$

- We say that $\underline{*}$ binds harder than $\underline{+}$.

## Operator Associativity

- The associativity of an operator describes how operators of equal precedence are grouped.
- $+$ and $-$ are usually left associative:

$$4 - 2 + 3$$

means

$$(4 - 2) + 3 = 5,$$

not

$$4 - (2 + 3) = -1.$$

We say that $+$ associates to the left.

- ˆ associates to the right:

$$2\hat{}3\hat{}4 = 2\hat{}(3\hat{}4).$$

## Expression Grammars

- We must write unambiguous expression grammars that reflect the associativity and precedence of all operators.
- The next slide gives the algorithm for writing such grammars.

Resulting Expression Grammar:

expr ::= expr <u>+</u> term | term

term ::= term <u>*</u> factor | factor

factor ::= <u>(</u> expr <u>)</u> | <u>`number`</u>

1. Create one non-terminal for each precedence level, for example $p_1, p_2, \cdots, p_n$, where $p_n$ has the highest precedence level.
2. For operator *op* at precedence level $i$ construct the following production if the operator is
   - left associative:

     $p_i$ ::= $p_i$ *op* $p_{i+1}$ | $p_{i+1}$
   - right associative:

     $p_i$ ::= $p_{i+1}$ *op* $p_i$ | $p_{i+1}$
3. Construct a production for nonterminal $p_{n+1}$ which represents primary expressions such as identifiers, numbers, parenthesized expressions, etc:

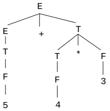
   $p_{n+1}$ ::= $(p_1)$ | num | id

E ::= E <u>+</u> T | T

T ::= T <u>*</u> F | F

F ::= <u>`number`</u>

$\boxed{5 + 4 * 3}$




$$
\begin{aligned}
E &\Rightarrow E + T \\
&\Rightarrow T + T \\
&\Rightarrow F + T \\
&\Rightarrow 5 + T \\
&\Rightarrow 5 + T * F \\
&\Rightarrow 5 + F * F \\
&\Rightarrow 5 + 4 * F \\
&\Rightarrow 5 + 4 * 3
\end{aligned}
$$

$$
\begin{aligned}
E &\Rightarrow E + T \\
&\Rightarrow E + T * F \\
&\Rightarrow E + T * 3 \\
&\Rightarrow E + F * 3 \\
&\Rightarrow E + 4 * 3 \\
&\Rightarrow T + 4 * 3 \\
&\Rightarrow F + 4 * 3 \\
&\Rightarrow 5 + 4 * 3
\end{aligned}
$$

# Top-Down Parsing

## Recursive Descent Parsing

```
PROCEDURE S ();
   IF curr_tok = if THEN
      match(if); E();
      match(then); S();
   ELSIF curr_tok = id THEN
      match(id); match(:=); E();
   ELSE syntax error ENDIF;
PROCEDURE E ();
   IF curr_tok = id THEN match(id);
   ELSE IF curr_tok = num THEN match(num);
   ELSE E(); match(+); E();
   ENDIF;
```

$$\begin{array}{lcl} S & \rightarrow & \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \\ & | & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\ E & \rightarrow & E\underline{\texttt{+}}E \\ & | & \underline{\texttt{id}} \mid \underline{\texttt{num}} \end{array}$$

## Recursive Descent—*Small Problem 1*

We may loop forever:

```
PROCEDURE E ();
   IF ···
   ELSE E(); match(+); E();
   ···
```

## Recursive Descent—*Small Problem 2*

What about productions that start out similarly:

$$\begin{array}{lcl} S & \rightarrow & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \mid \\ & & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ \underline{\texttt{else}}\ S \end{array}$$

```
PROCEDURE S ();
  IF curr_tok = if THEN
    match(if); E(); match(then); S();
  ELSIF curr_tok = if THEN
    match(if); E(); match(then);
    S(); match(else); S();
  ELSIF ··· ENDIF
```

## Recursive Descent—*Small Problem 3*

What if there are several possible "next" tokens:

$$\begin{array}{lcl} \text{prog} & \rightarrow & \text{decl} \mid \text{stat} \\ \text{stat} & \rightarrow & \underline{\texttt{if}}\cdots \mid \underline{\texttt{id()}} \mid \underline{\texttt{while}}\cdots \\ \text{decl} & \rightarrow & \underline{\texttt{int}}\ \underline{\texttt{id}} \mid \underline{\texttt{real}}\ \underline{\texttt{id}} \end{array}$$

```
PROCEDURE prog ();
   IF curr_tok ∈ {if,id,while} THEN stat();
   ELSIF curr_tok ∈ {int,real} THEN decl()
   ELSE syntax error ENDIF;
END;
PROCEDURE stat (); ··· END;
PROCEDURE decl (); ··· END;
```

## Left Recursion Removal

Left recursion must be removed from the grammar, by turning it into right recursion:

$$A \rightarrow A\alpha \mid \beta \quad \Rightarrow \quad \begin{array}{lcl} A & \rightarrow & \beta R \\ R & \rightarrow & \alpha R \mid \epsilon \end{array}$$

Example:

$$\begin{array}{rcl} \text{expr} & \rightarrow & \text{expr} \ \underline{+} \ \text{term} \mid \text{term} \\ & \Downarrow & \\ \text{expr} & \rightarrow & \text{term} \ R \\ R & \rightarrow & \underline{+} \ \text{term} \ R \mid \epsilon \end{array}$$

## Left Recursion Removal...

- After left recursion removal, our expression grammar

$$\begin{array}{lcl} E & \rightarrow & E \underline{+} T \mid T \\ T & \rightarrow & T \underline{*} F \mid F \\ F & \rightarrow & \underline{(} \ E \ \underline{)} \mid \underline{\text{id}} \end{array}$$

turns into

$$\begin{array}{lcl} E & \rightarrow & T \ E' \\ E' & \rightarrow & \underline{+} \ T \ E' \mid \epsilon \\ T & \rightarrow & F \ T' \\ T' & \rightarrow & \underline{*} \ F \ T' \mid \epsilon \\ F & \rightarrow & \underline{(} \ E \ \underline{)} \mid \underline{\text{id}} \end{array}$$

## Left Factoring

A top-down parser that reads input from left-to-right, can't choose between productions $E \rightarrow abF$ and $E \rightarrow abcF$. These must be left factored.

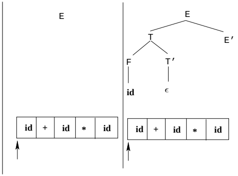
$$A \rightarrow \alpha\beta_1 \mid \alpha\beta_2 \quad \Rightarrow \quad \begin{array}{lcl} A & \rightarrow & \alpha A' \\ A' & \rightarrow & \beta_1 \mid \beta_2 \end{array}$$

Example:

$$\begin{array}{lcl} S & \rightarrow & \underline{\texttt{if}} \ E \ \underline{\texttt{then}} \ S \ \underline{\texttt{else}} \ S \mid \\ & & \underline{\texttt{if}} \ E \ \underline{\texttt{then}} \ S \end{array} \quad \Rightarrow \quad \begin{array}{lcl} S & \rightarrow & \underline{\texttt{if}} \ E \ \underline{\texttt{then}} \ S \ S' \\ S' & \rightarrow & \underline{\texttt{else}} \ S \mid \epsilon \end{array}$$

## Top-Down Expression Parser

$$\begin{array}{lcl} E & \rightarrow & T \ E' \\ E' & \rightarrow & \underline{+} \ T \ E' \mid \epsilon \\ T & \rightarrow & F \ T' \\ T' & \rightarrow & \underline{*} \ F \ T' \mid \epsilon \\ F & \rightarrow & \underline{(} \ E \ \underline{)} \mid \underline{\text{id}} \end{array}$$



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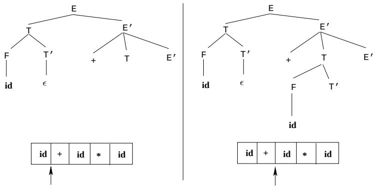


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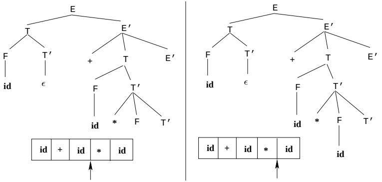
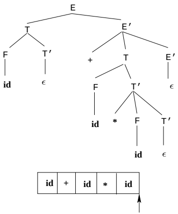



## Top-Down Expression Parser...




## Readings and References

- Read Louden, pp. 143–196.
- Or, the Dragon Book:
  Top-Down Parsing 181–190
  Error Recovery 192–195
  Recursive Descent Parsing 40–55, 75–76