CSc 453 — Compilers and Systems Software

6 : Top-Down Parsing I

Christian Collberg
Department of Computer Science
University of Arizona
collberg@gmail.com

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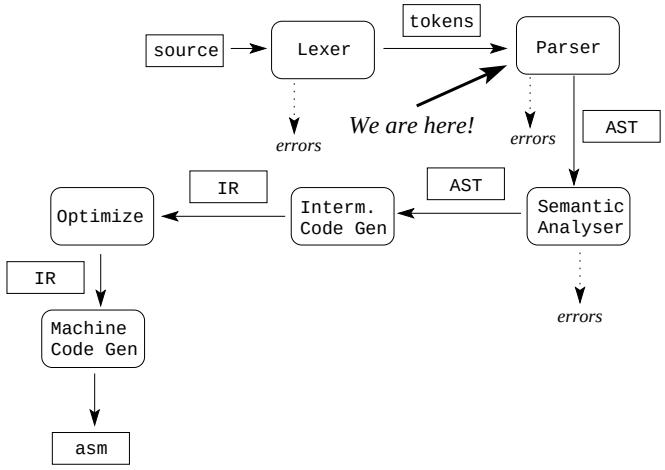
Overview

2 Compiler Phases



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Grammars

4 Context Free Grammars

- CFGs are used to describe the syntax of programming languages. A *production*

$$S \rightarrow \texttt{if}\ E\ \texttt{then}\ S_1\ \texttt{else}\ S_2$$

in a CFG says

"If S_1 and S_2 are statements and E an expression then '`if` E `then` S_1 `else` S_2' is a statement".

Notice that this production is *recursive*; it allows if-statements to occur within if-statements.

5 Context Free Grammars...

$$S \rightarrow \texttt{if}\ E\ \texttt{then}\ S_1\ \texttt{else}\ S_2$$

- `if`, `then`, and `else` are *terminal symbols* or *tokens*.
- S, S_1, S_2, and E are *non-terminals*. They are like "variables", that represent the kinds of strings that the grammar defines as *statements* or *expressions*, respectively.

6 CFG Notation

terminals:
`a`, `b`, `c`, $\cdots$, `+`, `-`, $\cdots$, `0`, `1`, $\cdots$, `if`, `do`.

nonterminals:
A, B, C, $\cdots$, S, $\cdots$, expr, stmt.

grammar symbols:
$X, Y, Z, \cdots$ (either terminals or nonterminals).

strings of terminals:
$u, v, w \cdots$.

strings of grammar symbols:
$\alpha, \beta, \gamma, \cdots$ (strings of terminals or nonterminals).

productions:
$A \rightarrow \alpha_1, A \rightarrow \alpha_2, \cdots, A \rightarrow \alpha_k$, or $A \rightarrow \alpha_1 \mid \alpha_2 \cdots \mid \alpha_k$.

7 Derivations — Productions as *Rewrite Rules*

1. Start with the *start symbol*, S.
2. Pick any production $S \rightarrow \alpha$, eg. $S \rightarrow$ `id :=` E.
3. We say that S *derives* `id :=` E, or $S \Rightarrow$ `id :=` E. '`id :=` E' is a *sentential form* derived from S.
4. Repeat: pick a nonterminal A from the sentential form, replace with the RHS of a production $A \rightarrow \alpha$: $S \Rightarrow$ `id :=` $E \Rightarrow$ `id :=` E`+`$E \Rightarrow$ `id :=` `id` $+ E \Rightarrow$ `id :=` `id+num`. $S \stackrel{*}{\Rightarrow}$ `id :=` `id+num`.

$$\begin{aligned} S &\rightarrow \texttt{id}\ \texttt{:=}\ E \mid \texttt{if}\ E\ \texttt{then}\ S \\ E &\rightarrow E\texttt{+}E \mid \texttt{id} \mid \texttt{num} \end{aligned}$$

8 Terminology

- A grammar is a 4-tuple

  (non-terminals, terminals, productions, start-symbol)

  or

  $$(N, \Sigma, P, S)$$

- A production is of the form $\alpha \rightarrow \beta$ where α, β are taken from $N \bigcup \Sigma$.
- Read $\alpha \rightarrow \beta$ as "rewrite α with β".
- Read $\Rightarrow$ as "directly derives".
- Read $\overset{r}{\Rightarrow}$ as "directly derives using rule r".
- Read $\overset{*}{\Rightarrow}$ as "derives in zero or more steps".

9 Derivations...

- $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if
  - $\boxed{A \rightarrow \gamma}$ is a production, and
  - α and β are strings of grammar symbols.

$\Rightarrow$: Derives in one step.

$\overset{*}{\Rightarrow}$: Derives in 0 or more steps.

$\overset{+}{\Rightarrow}$: Derives in 1 or more steps.

$\underset{lm}{\Rightarrow}$: Leftmost derivation.

$\underset{rm}{\Rightarrow}$: Rightmost derivation.

$L(G)$: The language generated by grammar G. This is the set of strings w, such that there is a derivation $S \overset{+}{\Rightarrow} w$, where S is G's start-symbol.

10 Derivations...

The string of terminal symbols $\boxed{\texttt{id:=id+num}}$ is generated by a leftmost derivation:

$$\begin{aligned} S &\underset{lm}{\Rightarrow} \texttt{id} \texttt{:=} E \underset{lm}{\Rightarrow} \texttt{id} \texttt{:=} E \texttt{+} E \\ &\underset{lm}{\Rightarrow} \texttt{id} \texttt{:=} \texttt{id+} E \underset{lm}{\Rightarrow} \texttt{id} \texttt{:=} \texttt{id+num} \\ S &\overset{+}{\Rightarrow} \texttt{id:=id+num} \end{aligned}$$

Example Grammar:

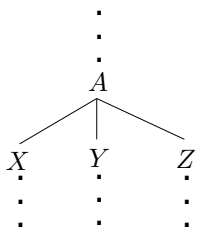
$$\begin{aligned} S &\rightarrow \texttt{id} \texttt{:=} E \mid \texttt{if} \; E \; \texttt{then} \; S \\ E &\rightarrow E \texttt{+} E \mid \texttt{id} \mid \texttt{num} \end{aligned}$$

11 Parse Trees...

- If one step of our derivation is

$$\cdots A \cdots \Rightarrow \cdots X\ Y\ Z \cdots$$

(i.e, we used the rule $A \to XYZ$) then we'll get a parse (sub-)tree



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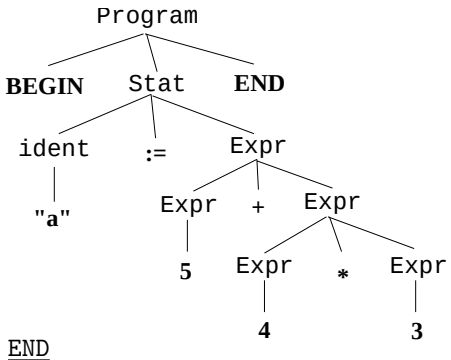

Program ⇒ BEGIN Stat END
⇒ BEGIN ident := Expr END
⇒ BEGIN "a" := Expr END
⇒ BEGIN "a" := Expr + Expr END
⇒ BEGIN "a" := 5 + Expr END
⇒ BEGIN "a" := 5 + Expr * Expr END
⇒ BEGIN "a" := 5 + 4 * Expr END
⇒ BEGIN "a" := 5 + 4 * 3 END

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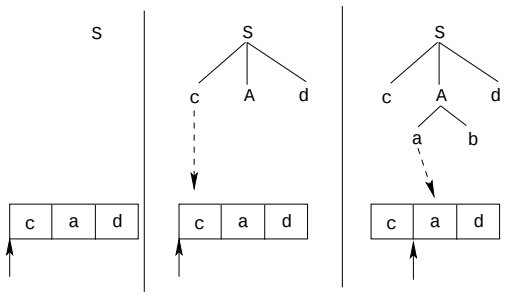
Top-Down Parsing

14 Top-Down Backtracking Parser

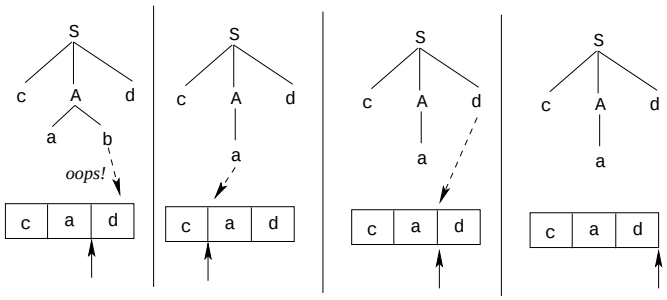
- Top-down parsing involves building a parse tree for the input string by starting at the root and adding nodes in preorder.

$$S \to cAd \qquad A \to ab|a$$



15 Top-Down Backtracking Parser...

- If a backtracking top-down parser chooses the wrong production rule to expand a node it backs up over the input, and undoes some of the parse tree construction:



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Grammar Rewriting

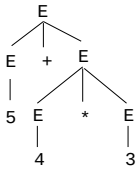
17 Ambiguous Grammars

- A grammar is ambiguous if some string of tokens can produce two (or more) different parse trees.

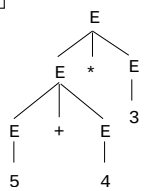
E ::= E + E | E * E | number

5 + 4 * 3

E ⇒ E + E
⇒ 5 + E
⇒ 5 + E * E
⇒ 5 + 4 * E
⇒ 5 + 4 * 3



E ⇒ E * E
⇒ E * 3
⇒ E + E * 3
⇒ E + 4 * 3
⇒ 5 + 4 * 3



18 Operator Precedence

- The *precedence* of an operator is a measure of its *binding power*, i.e. how strongly it attracts its operands.

- Usually $*$ has higher precedence than $+$:

  $$4 + 5 * 3$$

  means

  $$4 + (5 * 3),$$

  not

  $$(4 + 5) * 3.$$

- We say that $\underline{*}$ binds harder than $\underline{+}$.

19 Operator Associativity

- The *associativity* of an operator describes how operators of equal precedence are grouped.
- $+$ and $-$ are usually *left associative*:

  $$4 - 2 + 3$$

  means

  $$(4 - 2) + 3 = 5,$$

  not

  $$4 - (2 + 3) = -1.$$

  We say that $+$ *associates to the left.*
- `^` associates to the right:

  $$2\text{^}3\text{^}4 = 2\text{^}(3\text{^}4).$$

20 Expression Grammars

- We must write unambiguous expression grammars that reflect the associativity and precedence of all operators.
- The next slide gives the algorithm for writing such grammars.

Resulting Expression Grammar:

expr ::= expr + term | term
term ::= term * factor | factor
factor ::= (expr) | number

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1. Create one non-terminal for each precedence level, for example $p_1, p_2, \cdots, p_n$, where p_n has the highest precedence level.
2. For operator *op* at precedence level i construct the following production if the operator is
   - left associative:

     p_i ::= p_i *op* p_{i+1} | p_{i+1}

   - right associative:

     p_i ::= p_{i+1} *op* p_i | p_{i+1}

3. Construct a production for nonterminal p_{n+1} which represents *primary* expressions such as identifiers, numbers, parenthesized expressions, etc:

   p_{n+1} ::= (p_1) | num | id

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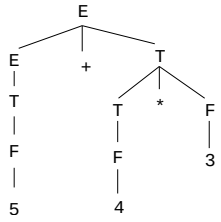
E ::= E + T | T
T ::= T * F | F
F ::= number

5 + 4 * 3

E ⇒ E + T
⇒ T + T
⇒ F + T
⇒ 5 + T
⇒ 5 + T * F
⇒ 5 + F * F
⇒ 5 + 4 * F
⇒ 5 + 4 * 3

E ⇒ E + T
⇒ E + T * F
⇒ E + T * 3
⇒ E + F * 3
⇒ E + 4 * 3
⇒ T + 4 * 3
⇒ F + 4 * 3
⇒ 5 + 4 * 3



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Top-Down Parsing

24 Recursive Descent Parsing

```
PROCEDURE S ();
   IF curr_tok = if THEN
      match(if); E();
      match(then); S();
   ELSIF curr_tok = id THEN
      match(id); match(:=); E();
   ELSE syntax error ENDIF;
PROCEDURE E ();
   IF curr_tok = id THEN match(id);
   ELSE IF curr_tok = num THEN match(num);
   ELSE E(); match(+); E();
   ENDIF;
```

$$
\begin{array}{rcl}
S & \rightarrow & \texttt{id} := E \\
 & | & \texttt{if}\ E\ \texttt{then}\ S \\
E & \rightarrow & E + E \\
 & | & \texttt{id} \mid \texttt{num}
\end{array}
$$

25 Recursive Descent—*Small Problem 1*

We may loop forever:

```
PROCEDURE E ();
   IF ···
   ELSE E(); match(+); E();
   ···
```

26 Recursive Descent—*Small Problem 2*

What about productions that start out similarly:

$$\begin{array}{rcl} S & \rightarrow & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ | \\ & & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ \underline{\texttt{else}}\ S \end{array}$$

```
PROCEDURE S ();
  IF curr_tok = if THEN
    match(if); E(); match(then); S();
  ELSIF curr_tok = if THEN
    match(if); E(); match(then);
    S(); match(else); S();
  ELSIF ··· ENDIF
```

27 Recursive Descent—*Small Problem 3*

What if there are several possible "next" tokens:

$$\begin{array}{rcl} \text{prog} & \rightarrow & \text{decl}\ |\ \text{stat} \\ \text{stat} & \rightarrow & \underline{\texttt{if}}\cdots\ |\ \underline{\texttt{id()}}\ |\ \underline{\texttt{while}}\cdots \\ \text{decl} & \rightarrow & \underline{\texttt{int}}\ \underline{\texttt{id}}\ |\ \underline{\texttt{real}}\ \underline{\texttt{id}} \end{array}$$

```
PROCEDURE prog ();
   IF curr_tok ∈ {if, id, while} THEN stat();
   ELSIF curr_tok ∈ {int, real} THEN decl()
   ELSE syntax error ENDIF;
END;
PROCEDURE stat (); ··· END;
PROCEDURE decl (); ··· END;
```

28 Left Recursion Removal

Left recursion must be removed from the grammar, by turning it into *right recursion*:

$$A \rightarrow A\,\alpha\ |\ \beta \quad \Rightarrow \quad \begin{array}{rcl} A & \rightarrow & \beta \\ R & \rightarrow & \alpha \end{array}$$

Example:

$$\begin{array}{rcl} \text{expr} & \rightarrow & \text{expr}\ \underline{+}\ \text{term}\ |\ \text{term} \\ & \Downarrow & \\ \text{expr} & \rightarrow & \text{term}\ R \\ R & \rightarrow & \underline{+}\ \text{term}\ R\ |\ \epsilon \end{array}$$

29 Left Recursion Removal...

- After left recursion removal, our expression grammar

$$\begin{array}{rcl} E & \rightarrow & E\underline{+}T|T \\ T & \rightarrow & T\underline{*}F|F \\ F & \rightarrow & \underline{(}\ E\ \underline{)}\ |\ \underline{\texttt{id}} \end{array}$$

turns into

$$\begin{array}{rcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & \underline{+}\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & \underline{*}\ F\ T' \mid \epsilon \\ F & \rightarrow & \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}} \end{array}$$

30 Left Factoring

A top-down parser that reads input from left-to-right, can't choose between productions $E \rightarrow abF$ and $E \rightarrow abcF$. These must be *left factored*.

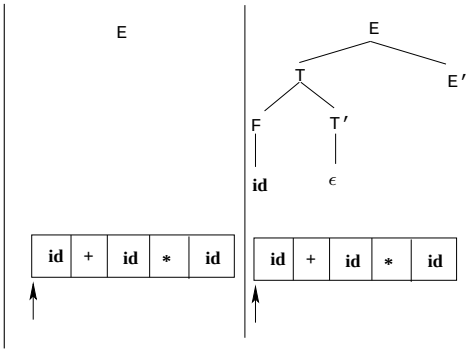
$$\begin{array}{rcl} A & \rightarrow & \alpha\ \beta_1 \mid \alpha\ \beta_2 \end{array} \qquad \Rightarrow \qquad \begin{array}{rcl} A & \rightarrow & \alpha\ A' \\ A' & \rightarrow & \beta_1 \mid \beta_2 \end{array}$$

Example:

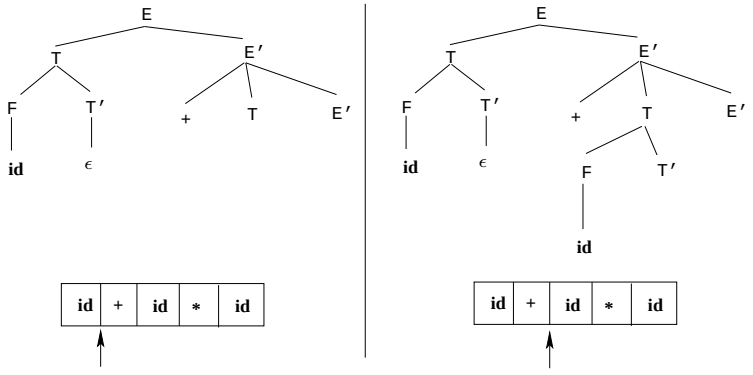
$$\begin{array}{rcl} S & \rightarrow & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ \underline{\texttt{else}}\ S \mid \\ & & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \end{array} \qquad \Rightarrow \qquad \begin{array}{rcl} S & \rightarrow & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ S' \\ S' & \rightarrow & \underline{\texttt{else}}\ S \mid \epsilon \end{array}$$

31 Top-Down Expression Parser

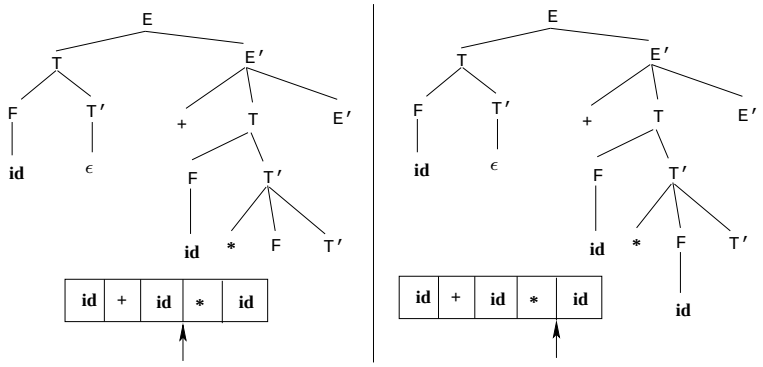
$$\begin{array}{rcl} E & \rightarrow & T\ E' \\ E' & \rightarrow & \underline{+}\ T\ E' \mid \epsilon \\ T & \rightarrow & F\ T' \\ T' & \rightarrow & \underline{*}\ F\ T' \mid \epsilon \\ F & \rightarrow & \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}} \end{array}$$



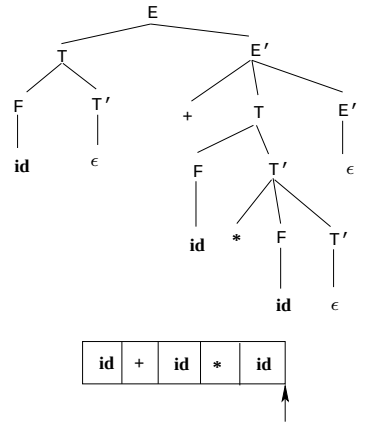
32 Top-Down Expression Parser...



33 Top-Down Expression Parser...



34 Top-Down Expression Parser...



35 Readings and References

- Read Louden, pp. 143–196.
- Or, the Dragon Book:

  Top-Down Parsing 181–190
  Error Recovery 192–195
  Recursive Descent Parsing 40–55, 75–76