CSc 453

Compilers and Systems Software

6 : Top-Down Parsing I

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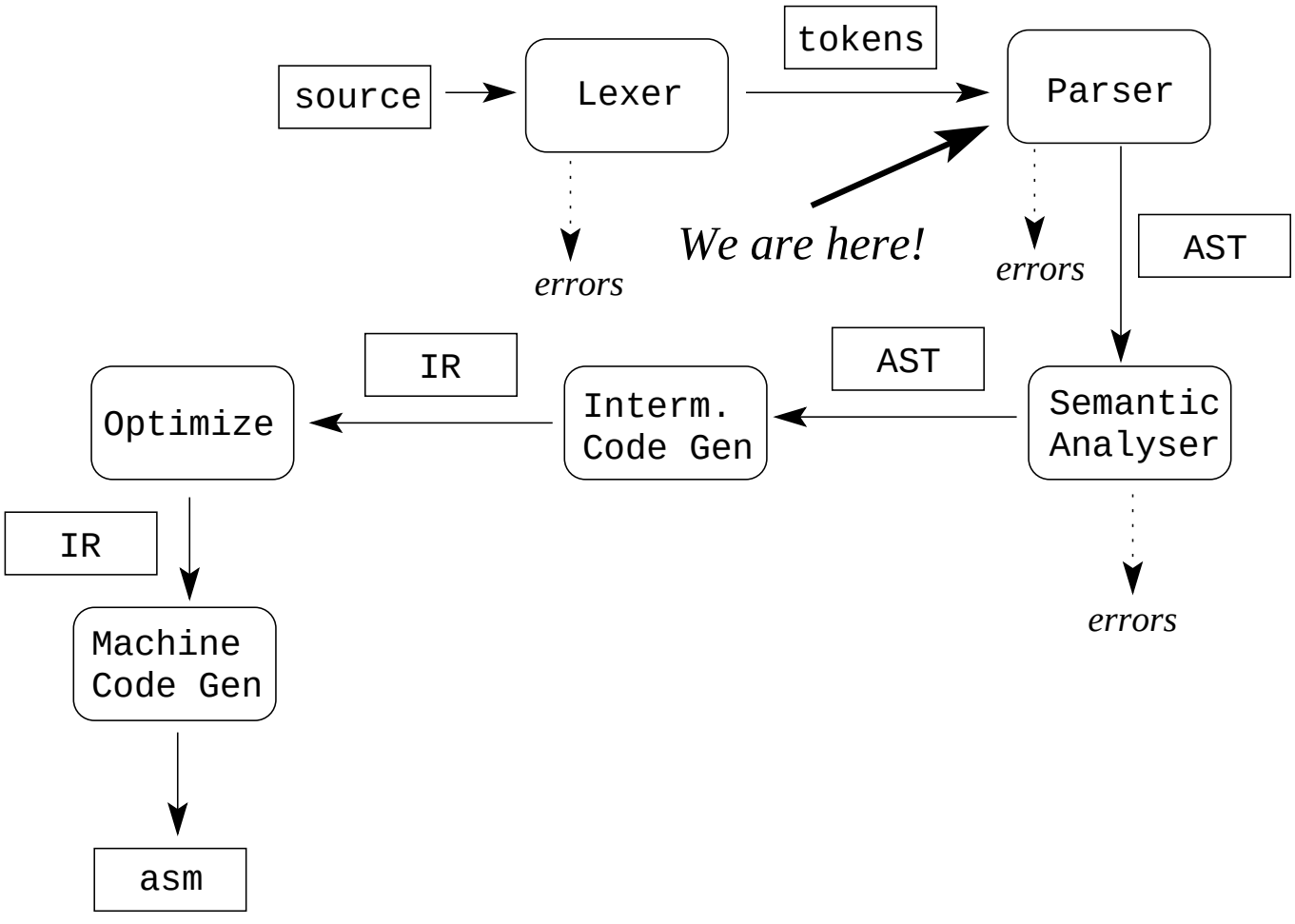
Overview

Compiler Phases



Grammars

Context Free Grammars

- CFGs are used to describe the syntax of programming languages. A production

$$S \rightarrow \texttt{if}\ E\ \texttt{then}\ S_1\ \texttt{else}\ S_2$$

in a CFG says

> *"If S_1 and S_2 are statements and E an expression then '$\texttt{if}\ E\ \texttt{then}\ S_1\ \texttt{else}\ S_2$' is a statement".*

Notice that this production is recursive; it allows if-statements to occur within if-statements.

Context Free Grammars. . .

$$S \rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S_1\ \underline{\texttt{else}}\ S_2$$

- $\underline{\texttt{if}}$, $\underline{\texttt{then}}$, and $\underline{\texttt{else}}$ are terminal symbols or tokens.
- S, S_1, S_2, and E are non-terminals. They are like "variables", that represent the kinds of strings that the grammar defines as statements or expressions, respectively.

CFG Notation

terminals:

$\underline{\texttt{a}}, \underline{\texttt{b}}, \underline{\texttt{c}}, \cdots, \underline{+}, \underline{-}, \cdots, \underline{0}, \underline{1}, \cdots, \underline{\texttt{if}}, \underline{\texttt{do}}$.

nonterminals:

$A, B, C, \cdots, S, \cdots$, expr, stmt.

grammar symbols:

$X, Y, Z, \cdots$ (either terminals or nonterminals).

strings of terminals:

$u, v, w \cdots$.

strings of grammar symbols:

$\alpha, \beta, \gamma, \cdots$ (strings of terminals or nonterminals).

productions:

$A \to \alpha_1, A \to \alpha_2, \cdots, A \to \alpha_k$, or
$A \to \alpha_1 \mid \alpha_2 \cdots \mid \alpha_k$.

Derivations — Productions as *Rewrite Rules*

1. Start with the start symbol, S.
2. Pick any production $S \to \alpha$, eg. $S \to \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$.
3. We say that S derives $\underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$, or $S \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$. '$\underline{\texttt{id}}\ \underline{\texttt{:=}}\ E$' is a sentential form derived from S.
4. Repeat: pick a nonterminal A from the sentential form, replace with the RHS of a production $A \to \alpha$:
   $S \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E\underline{\texttt{+}}E \Rightarrow$
   $\underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id}} + E \Rightarrow \underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id+num}}$. $S \overset{*}{\Rightarrow} \underline{\texttt{id}}\ \underline{\texttt{:=}}\ \underline{\texttt{id+num}}$.

$$
\begin{aligned}
S &\to \underline{\texttt{id}}\ \underline{\texttt{:=}}\ E \mid \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\
E &\to E\underline{\texttt{+}}E \mid \underline{\texttt{id}} \mid \underline{\texttt{num}}
\end{aligned}
$$

Terminology

- A grammar is a 4-tuple

  (non-terminals, terminals, productions, start-symbol)

  or

  $$(N, \Sigma, P, S)$$

- A production is of the form $\alpha \rightarrow \beta$ where α, β are taken from $N \bigcup \Sigma$.
- Read $\alpha \rightarrow \beta$ as "rewrite α with β".
- Read $\Rightarrow$ as "directly derives".
- Read $\overset{r}{\Rightarrow}$ as "directly derives using rule r".
- Read $\overset{*}{\Rightarrow}$ as "derives in zero or more steps".

Derivations...

- $\alpha A \beta \Rightarrow \alpha \gamma \beta$ if
  - $\boxed{A \rightarrow \gamma}$ is a production, and
  - α and β are strings of grammar symbols.

$\Rightarrow$: Derives in one step.

$\overset{*}{\Rightarrow}$: Derives in 0 or more steps.

$\overset{+}{\Rightarrow}$: Derives in 1 or more steps.

$\underset{lm}{\Rightarrow}$: Leftmost derivation.

$\underset{rm}{\Rightarrow}$: Rightmost derivation.

$L(G)$: The language generated by grammar G. This is the set of strings w, such that there is a derivation $S \overset{+}{\Rightarrow} w$, where S is G's start-symbol.

Derivations...

The string of terminal symbols $\boxed{\texttt{id:=id+num}}$ is generated by a leftmost derivation:

$$
\begin{aligned}
S &\overset{lm}{\Rightarrow} \texttt{id} := E \overset{lm}{\Rightarrow} \texttt{id} := E+E \\
&\overset{lm}{\Rightarrow} \texttt{id} := \texttt{id}+E \overset{lm}{\Rightarrow} \texttt{id} := \texttt{id+num} \\
S &\overset{+}{\Rightarrow} \texttt{id:=id+num}
\end{aligned}
$$

Example Grammar:

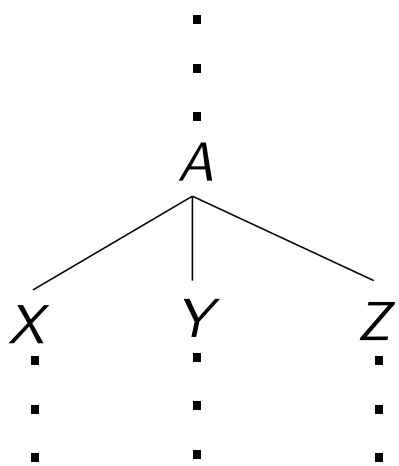
$$
\begin{aligned}
S &\rightarrow \texttt{id} := E \mid \texttt{if}\ E\ \texttt{then}\ S \\
E &\rightarrow E+E \mid \texttt{id} \mid \texttt{num}
\end{aligned}
$$

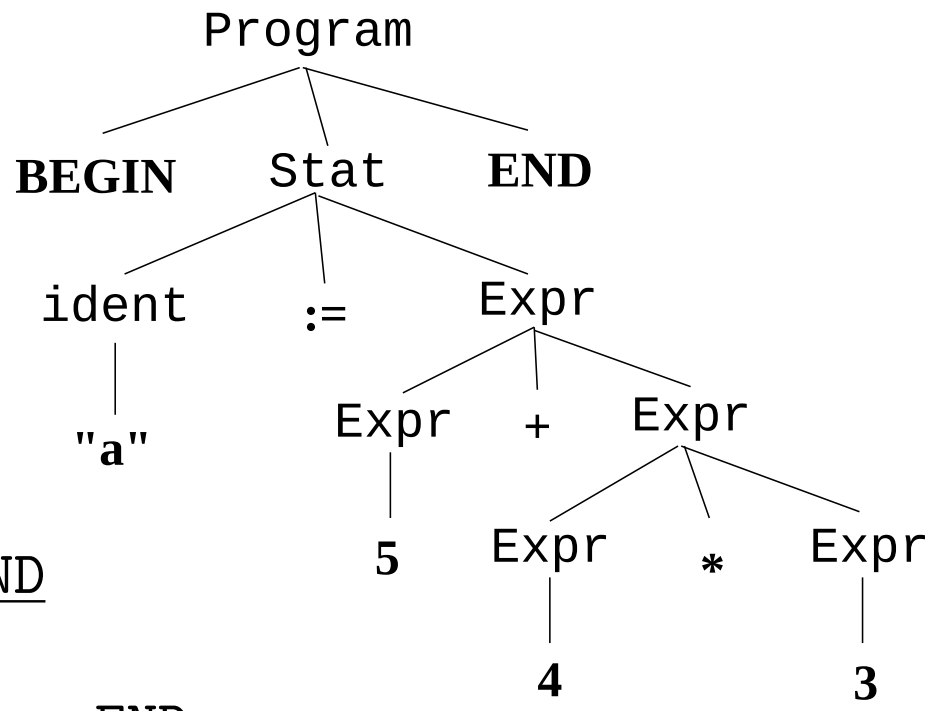
Parse Trees...

- If one step of our derivation is

$$\cdots A \cdots \Rightarrow \cdots X\ Y\ Z \cdots$$

(i.e, we used the rule $A \rightarrow XYZ$) then we'll get a parse (sub-)tree





Program $\Rightarrow$ BEGIN Stat END

$\Rightarrow$ BEGIN ident := Expr END

$\Rightarrow$ BEGIN "*a*" := Expr END

$\Rightarrow$ BEGIN "*a*" := Expr + Expr END

$\Rightarrow$ BEGIN "*a*" := 5 + Expr END

$\Rightarrow$ BEGIN "*a*" := 5 + Expr * Expr END

$\Rightarrow$ BEGIN "*a*" := 5 + 4 * Expr END

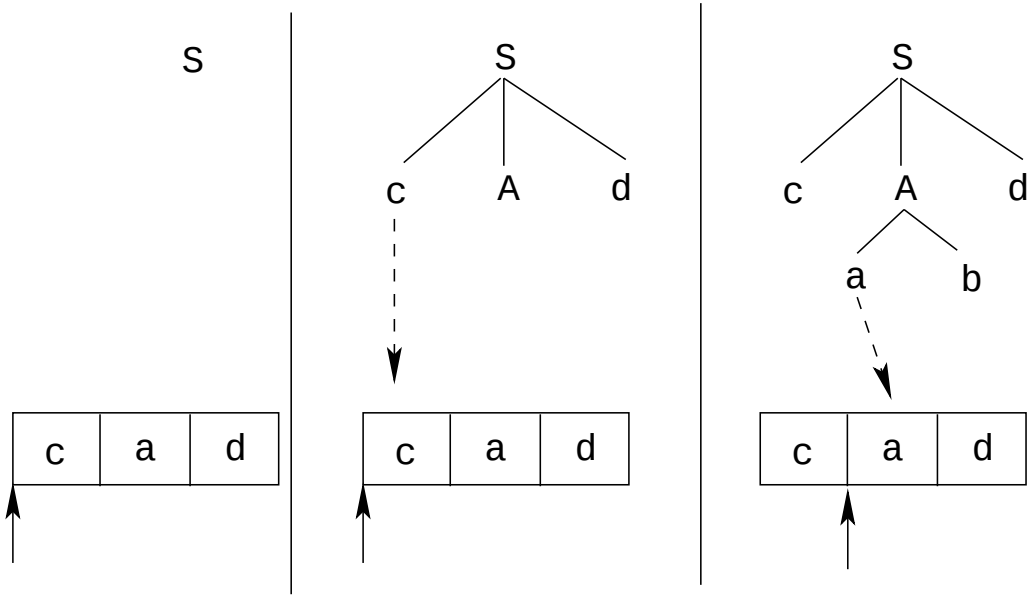
$\Rightarrow$ BEGIN "*a*" := 5 + 4 * 3 END

Top-Down Parsing

Top-Down Backtracking Parser

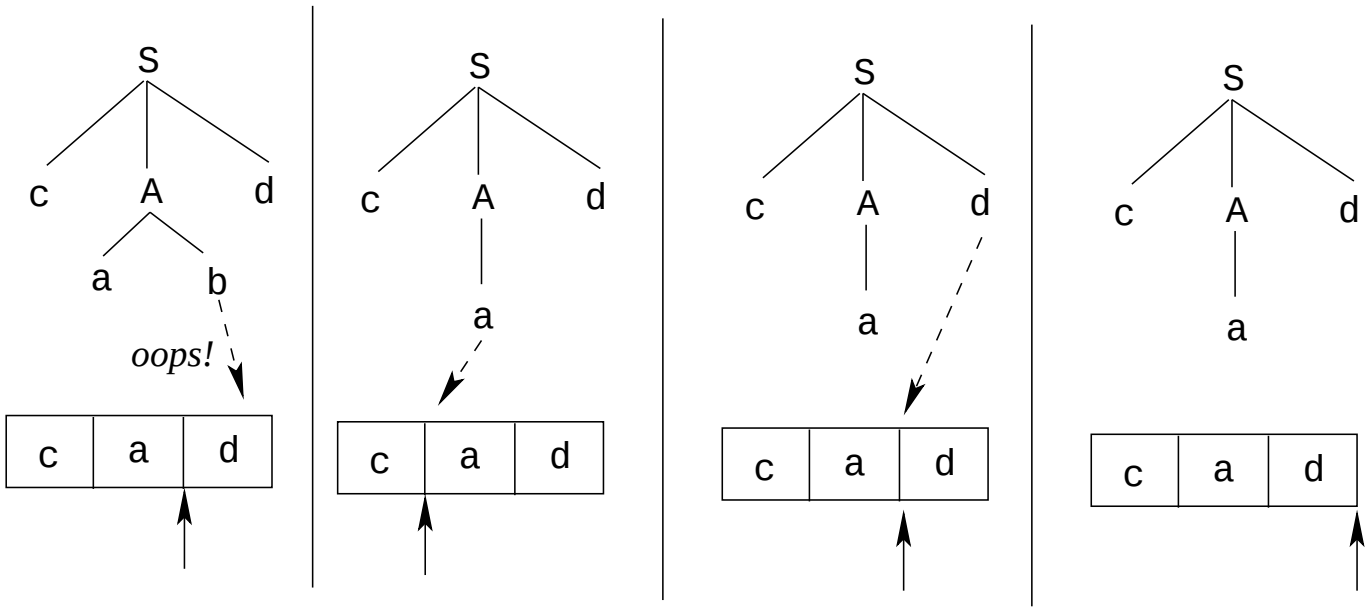
- Top-down parsing involves building a parse tree for the input string by starting at the root and adding nodes in preorder.

$$S \rightarrow cAd \qquad A \rightarrow ab|a$$



Top-Down Backtracking Parser...

- If a backtracking top-down parser chooses the wrong production rule to expand a node it backs up over the input, and undoes some of the parse tree construction:



Grammar Rewriting

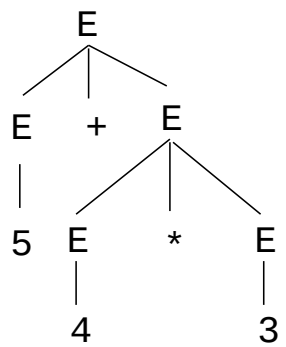
Ambiguous Grammars

- A grammar is ambiguous if some string of tokens can produce two (or more) different parse trees.

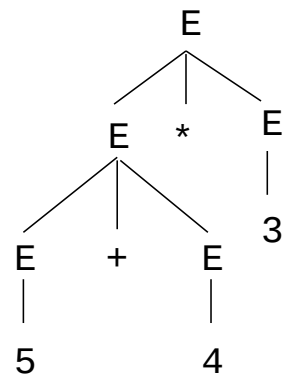
$$E ::= E + E \mid E * E \mid \texttt{number}$$

$$\boxed{5 + 4 * 3}$$

$$\begin{aligned} E &\Rightarrow E + E \\ &\Rightarrow 5 + E \\ &\Rightarrow 5 + E * E \\ &\Rightarrow 5 + 4 * E \\ &\Rightarrow 5 + 4 * 3 \end{aligned}$$



$$\begin{aligned} E &\Rightarrow E * E \\ &\Rightarrow E * 3 \\ &\Rightarrow E + E * 3 \\ &\Rightarrow E + 4 * 3 \\ &\Rightarrow 5 + 4 * 3 \end{aligned}$$



Operator Precedence

- The precedence of an operator is a measure of its binding power, i.e. how strongly it attracts its operands.
- Usually $*$ has higher precedence than $+$:

$$4 + 5 * 3$$

means

$$4 + (5 * 3),$$

not

$$(4 + 5) * 3.$$

- We say that $\underline{*}$ binds harder than $\underline{+}$.

Operator Associativity

- The associativity of an operator describes how operators of equal precedence are grouped.
- $+$ and $-$ are usually left associative:

$$4 - 2 + 3$$

means

$$(4 - 2) + 3 = 5,$$

not

$$4 - (2 + 3) = -1.$$

We say that $+$ associates to the left.

- ^ associates to the right:

$$2\text{^}3\text{^}4 = 2\text{^}(3\text{^}4).$$

Expression Grammars

- We must write unambiguous expression grammars that reflect the associativity and precedence of all operators.
- The next slide gives the algorithm for writing such grammars.

Resulting Expression Grammar:

expr ::= expr + term — term

term ::= term * factor — factor

factor ::= (expr) — `number`

1. Create one non-terminal for each precedence level, for example $p_1, p_2, \cdots, p_n$, where p_n has the highest precedence level.
2. For operator *op* at precedence level i construct the following production if the operator is
   - left associative:

   $p_i ::= p_i \ op \ p_{i+1} \mid p_{i+1}$
   - right associative:

   $p_i ::= p_{i+1} \ op \ p_i \mid p_{i+1}$
3. Construct a production for nonterminal p_{n+1} which represents primary expressions such as identifiers, numbers, parenthesized expressions, etc:

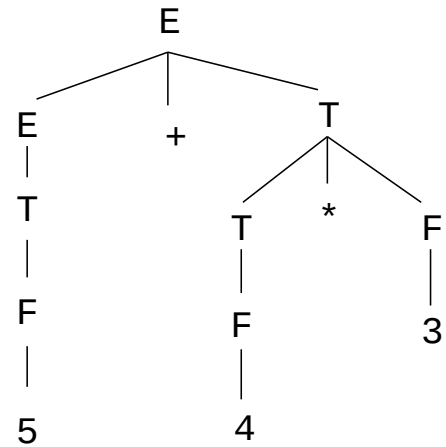
   $p_{n+1} ::= (p_1) \mid \text{num} \mid \text{id}$

5 + 4 * 3

E ::= E + T — T

T ::= T * F — F

F ::= number



E ⇒ E + T
⇒ T + T
⇒ F + T
⇒ 5 + T
⇒ 5 + T * F
⇒ 5 + F * F
⇒ 5 + 4 * F
⇒ 5 + 4 * 3

E ⇒ E + T
⇒ E + T * F
⇒ E + T * 3
⇒ E + F * 3
⇒ E + 4 * 3
⇒ T + 4 * 3
⇒ F + 4 * 3
⇒ 5 + 4 * 3

Top-Down Parsing

Recursive Descent Parsing

```
PROCEDURE S ();
    IF curr_tok = if THEN
       match(if); E();
       match(then); S();
    ELSIF curr_tok = id THEN
       match(id); match(:=); E();
    ELSE syntax error ENDIF;
PROCEDURE E ();
    IF curr_tok = id THEN match(id);
    IF curr_tok = num THEN match(num);
    ELSE E(); match(+); E();
    ENDIF;
```

$$
\begin{array}{lcl}
S & \rightarrow & \underline{\texttt{id}}\ \underline{:=}\ E \\
 & | & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \\
E & \rightarrow & E \underline{+} E \\
 & | & \underline{\texttt{id}} \mid \underline{\texttt{num}}
\end{array}
$$

Recursive Descent—*Small Problem 1*

We may loop forever:

```
PROCEDURE E ();
    IF ···
    ELSE E(); match(+); E();
    ...
```

Recursive Descent—*Small Problem 2*

What about productions that start out similarly:

$$\begin{aligned} S \quad \rightarrow \quad & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \mid \\ & \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ \underline{\texttt{else}}\ S \end{aligned}$$

```
PROCEDURE S ();
  IF curr_tok = if THEN
    match(if); E(); match(then); S();
  ELSIF curr_tok = if THEN
    match(if); E(); match(then);
    S(); match(else); S();
  ELSIF ··· ENDIF
```

Recursive Descent—*Small Problem 3*

What if there are several possible "next" tokens:

$$\begin{array}{rcl} \text{prog} & \longrightarrow & \text{decl} \mid \text{stat} \\ \text{stat} & \longrightarrow & \underline{\texttt{if}} \cdots \mid \underline{\texttt{id()}} \mid \underline{\texttt{while}} \cdots \\ \text{decl} & \longrightarrow & \underline{\texttt{int}}\ \underline{\texttt{id}} \mid \underline{\texttt{real}}\ \underline{\texttt{id}} \end{array}$$

```
PROCEDURE prog ();
    IF curr_tok ∈ {if, id, while} THEN stat();
    ELSIF curr_tok ∈ {int, real} THEN decl()
    ELSE syntax error ENDIF;
END;
PROCEDURE stat (); ··· END;
PROCEDURE decl (); ··· END;
```

Left Recursion Removal

Left recursion must be removed from the grammar, by turning it into right recursion:

$$A \rightarrow A\,\alpha \mid \beta \quad \Rightarrow \quad \begin{aligned} A &\rightarrow \beta\,R \\ R &\rightarrow \alpha\,R \mid \epsilon \end{aligned}$$

Example:

$$\begin{aligned} \text{expr} &\rightarrow \text{expr} \pm \text{term} \mid \text{term} \\ &\Downarrow \\ \text{expr} &\rightarrow \text{term}\ R \\ R &\rightarrow \pm\ \text{term}\ R \mid \epsilon \end{aligned}$$

Left Recursion Removal...

- After left recursion removal, our expression grammar

$$
\begin{aligned}
E &\rightarrow E\underline{+}T \mid T \\
T &\rightarrow T\underline{*}F \mid F \\
F &\rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}}
\end{aligned}
$$

turns into

$$
\begin{aligned}
E &\rightarrow T\ E' \\
E' &\rightarrow \underline{+}\ T\ E' \mid \epsilon \\
T &\rightarrow F\ T' \\
T' &\rightarrow \underline{*}\ F\ T' \mid \epsilon \\
F &\rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}}
\end{aligned}
$$

Left Factoring

A top-down parser that reads input from left-to-right, can't choose between productions $E \rightarrow abF$ and $E \rightarrow abcF$. These must be left factored.

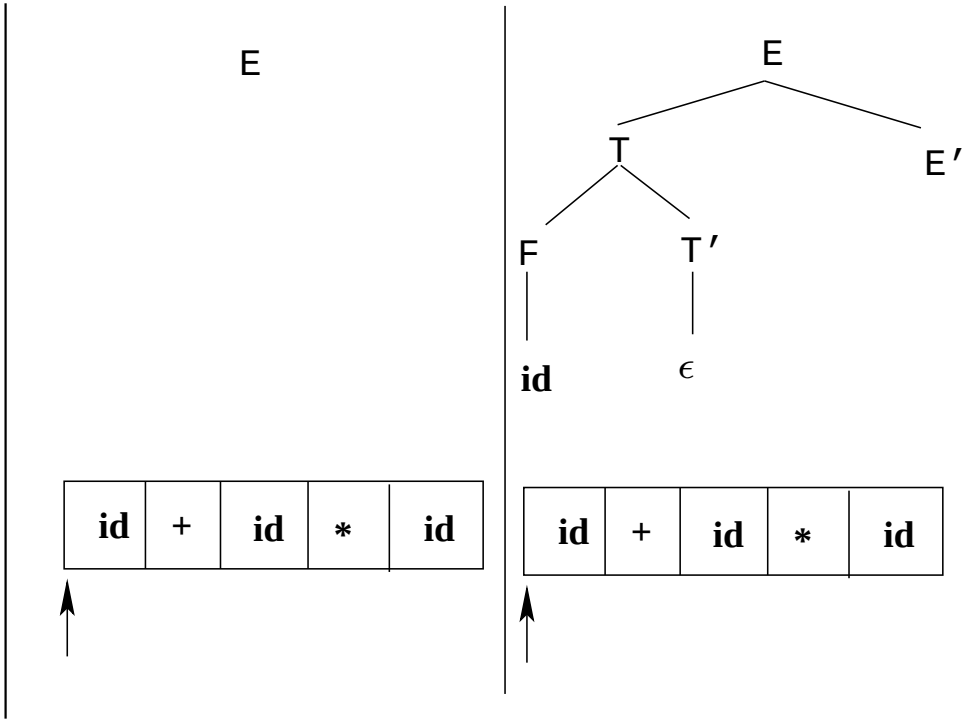
$$A \rightarrow \alpha\,\beta_1 \mid \alpha\,\beta_2 \quad \Rightarrow \quad \begin{aligned} A &\rightarrow \alpha\,A' \\ A' &\rightarrow \beta_1 \mid \beta_2 \end{aligned}$$

Example:

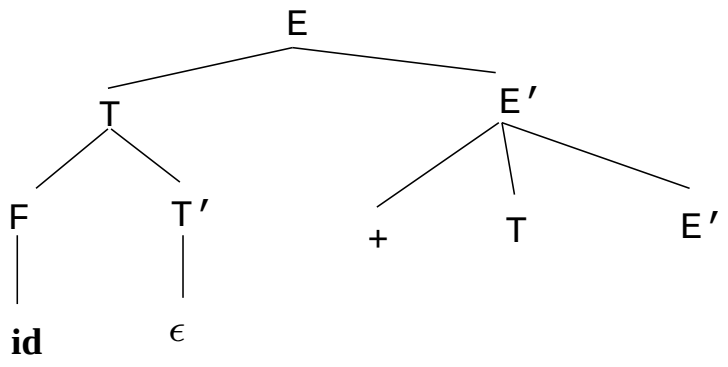
$$\begin{aligned} S &\rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ \underline{\texttt{else}}\ S \mid \\ &\quad\ \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S \end{aligned} \quad \Rightarrow \quad \begin{aligned} S &\rightarrow \underline{\texttt{if}}\ E\ \underline{\texttt{then}}\ S\ S' \\ S' &\rightarrow \underline{\texttt{else}}\ S \mid \epsilon \end{aligned}$$

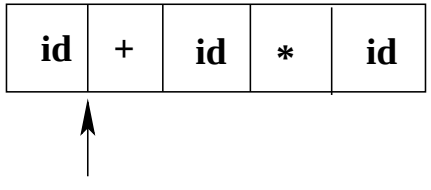
Top-Down Expression Parser

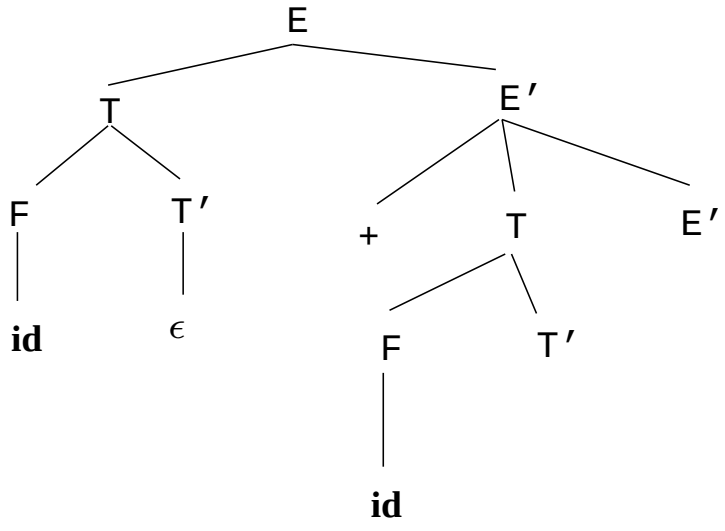
$$
\begin{aligned}
E &\rightarrow T\ E' \\
E' &\rightarrow \underline{+}\ T\ E' \mid \epsilon \\
T &\rightarrow F\ T' \\
T' &\rightarrow \underline{*}\ F\ T' \mid \epsilon \\
F &\rightarrow \underline{(}\ E\ \underline{)} \mid \underline{\texttt{id}}
\end{aligned}
$$

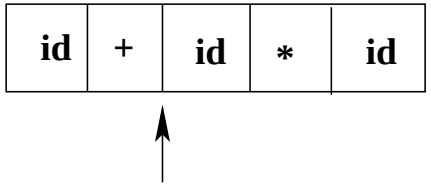


Top-Down Expression Parser...

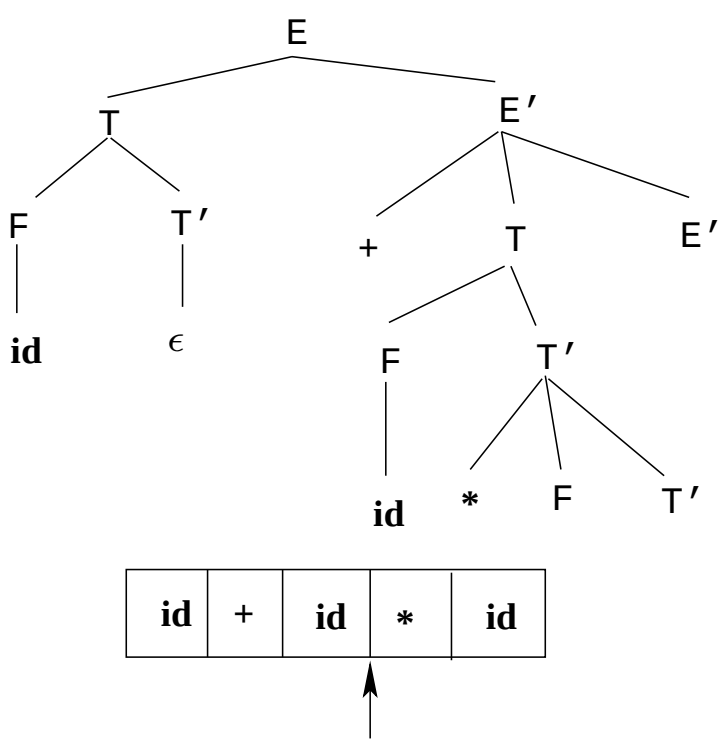


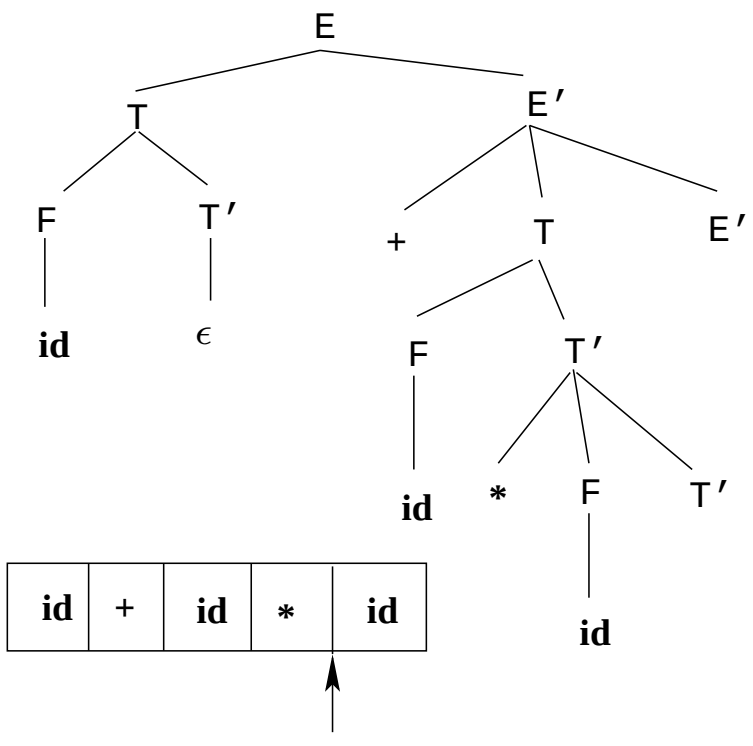




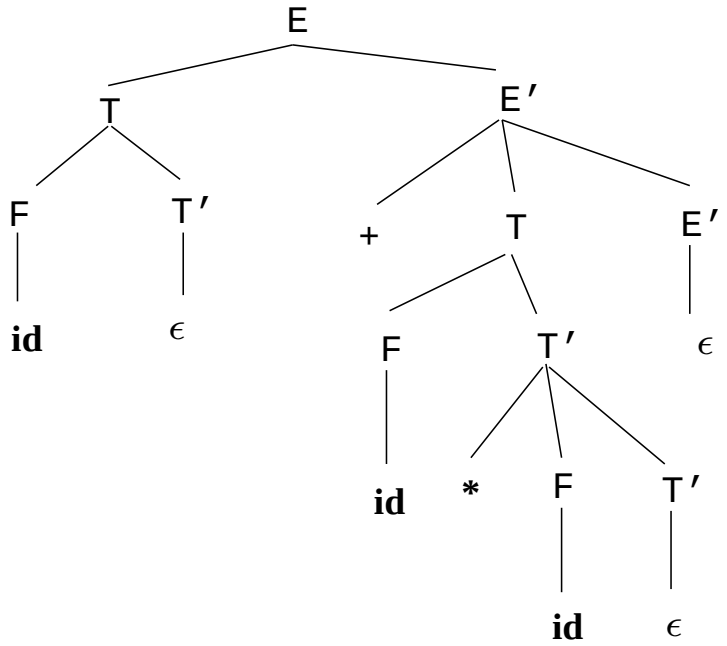


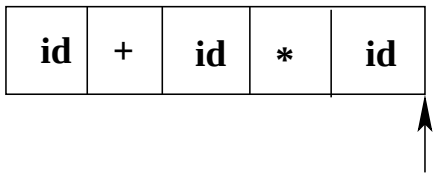
Top-Down Expression Parser...





Top-Down Expression Parser...





Readings and References

- Read Louden, pp. 143–196.
- Or, the Dragon Book:

  Top-Down Parsing 181–190
  Error Recovery 192–195
  Recursive Descent Parsing 40–55, 75–76