CSc 553

Principles of Compilation

28 : Optimization III

Department of Computer Science
University of Arizona

collberg@gmail.com

Introduction

Computing Data-Flow Info.

- There are two principal methods of solving data-flow problems:
  1. Let gen,kill,in,out be AST attributes and the data-flow equations attribute evaluation rules. We'll look at this later.
  2. Treat data-flow equations as recurrences, and iterate over the set of equations until a solution is found.
- Sets are stored as bit-vectors, with one element for each possible object.

$$\begin{array}{rcl} \texttt{in[B1]} & = & \{d_3, d_5, d_7\} \\ & \equiv & \begin{array}{c|c|c|c|c|c|c|c} 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ d_1 & d_2 & d_3 & d_4 & d_5 & d_6 & d_7 & d_8 \end{array} \end{array}$$

Data-flow Analysis by Iteration

Reaching Definitions — Equations

$$\texttt{out}[B] = \texttt{gen}[B] \cup (\texttt{in}[B] - \texttt{kill}[B])$$
$$\texttt{in}[B] = \bigcup_{\texttt{preds}\,P\ \texttt{of}\ B} \texttt{out}[P]$$

- A definition $d : a := b + c$ reaches a use of a at point p, if the value given to a at d could be used at p.
- gen[B] is the set of definitions generated within B, that reach the end of B.
- kill[B] is the set of definitions outside B, killed by definitions within B.
- in[B] is the set of definitions valid at the entrance to B, out[B] is those valid at the exit of B.
- The equations for in and out are valid for each basic block.

Iterative Algorithms I

1. Compute gen and kill for each block.
2. Set up the $2n$ in- and out-equations for the n basic blocks, and set in[B]=out[B]={} for each block B.
3. Repeat until no more changes:
   - For each block B eval. in[B] & out[B].

Formal Algorithm:

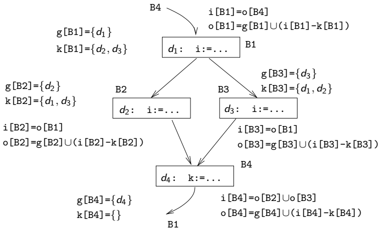
FOR each block B **DO**
  out[B] := in[B] := { };
END;
WHILE any out[B] has changed **DO**
  FOR each block B **DO**
    in[B] := $\bigcup_{\texttt{preds}\,P\ \texttt{of}\ B}$ out[P]
    out[B] := $\texttt{gen}[B] \cup (\texttt{in}[B] - \texttt{kill}[B])$
  END;
END

Iterative Alg. Example I (a)

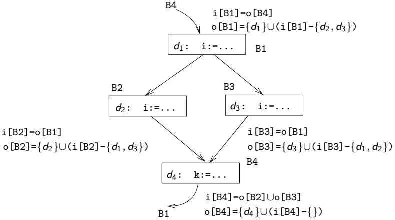
```
REPEAT
  d1: i := ...;
      IF ...THEN
  d2:   i := ...
      ELSE
  d3:   i := ...
      ENDIF;
  d4: k := ...
UNTIL ...;
```

- Start by setting up the $2n$ in- and out-equations (slide (b)).
- Simplify the example by inlining gen and kill into the equations for in and out (slide (c)).
- Visit each block in turn (we use numerical order, B1, B2, B3, B4) and evaluate in and out (slides (d)–(g)).

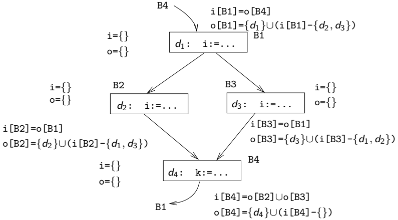
Iterative Alg. Example I (b)



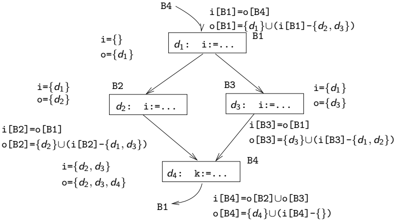
Iterative Alg. Example I (c) – Simplified



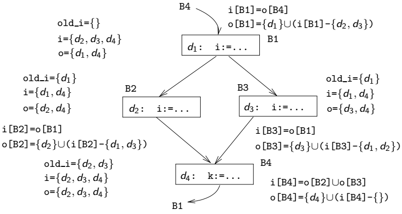
Iterative Alg. Example I (d) – Initialized



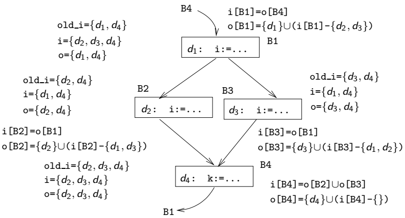
Iterative Alg. Example I (e) – 1st Iteration



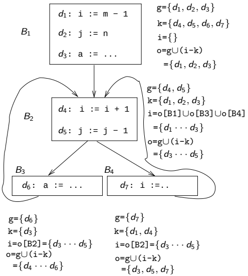
Iterative Alg. Example I (f) – 2nd Iteration



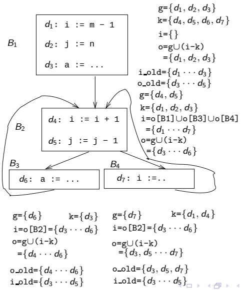
Iterative Alg. Example I (g) – 3rd Iteration



Example II (a) – 1st Iteration



Example II (b) – 2nd Iteration



Live Variable Analysis

Live Variable Analysis I

- For each definition/use of a variable V, **Global Live Variable Analysis** answers the question

  "Could the value of V *computed/used here be used further on in the program?"*

- If a variable V is stored in a register R5 and V is dead at the end of the block, then we don't have to store R5 back into V.
- Assignments to dead variables can be removed.

```
R5 := V;
R5 := R5 + 1;
```

V (stored in R5) is incremented

V is dead here.

No further used of V here.

Live Variable Analysis II

in[B] Variables live on entrance to B.

out[B] Variables live on exit from B.

def[B] Variables assigned values in B before the variable is used:

```
B := ... C ...;
C := ...;
... := ... B ...;
```

def={ B }
C is not in def since its value is being used before C is assigned to.

use[B] Variables whose values are used before being assigned to:

```
B := ... C ...;
C := ...;
... := ... B ...;
```

use={ C }
B is not in use since it is assigned a value before being used.

Live Variable Analysis III

Data-Flow (Equations):

$$\mathtt{in}[\mathrm{B}] = \mathtt{use}[\mathrm{B}] \cup (\mathtt{out}[\mathrm{B}] - \mathtt{def}[\mathrm{B}])$$

Data-Flow (English):

- V is live at the entrance to B if
  1. it is being used before it's defined (i.e. $V \in \mathtt{use}[\mathrm{B}]$)

     in={...C...} use={ C }

     ```
     B := ... C ...;
     C := ...;
     ```

     C is in in[B] since it's value is used before C is defined.

  2. it is live at the exit of the block, and it is not defined within the block (i.e. $V \in \mathtt{out}[\mathrm{B}]$ and $V \notin \mathtt{def}[\mathrm{B}]$)

     in={...C...}

     ```
     B := ... ;
     ```

     def={ B }

     out={...C,B...}

Live Variable Analysis IV

Data-Flow (Equations):

$$\mathtt{out}[\mathrm{B}] = \bigcup_{\substack{\texttt{successors} \\ S \text{ of } B}} \mathtt{in}[\mathrm{S}]$$

Data-Flow (English):

- A variable V is live coming out of B if it is live going into any one of B's successors.

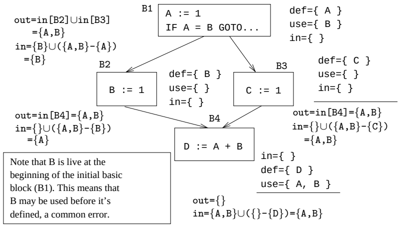
B
```
C := ...
```
out={ C }

in={}

in={ C }

```
...:=...C...
```

Live Variable Analysis V – Example



Live Variables VI – Example

`out[B4]={}` since out is the union of all of B4's successor's in, and B4 doesn't have any successors.

`in[B4]={}` because both A & B are live coming in to B4, i.e. their values will be used before they are assigned new values.

`out[B3]=in[B4]={A,B}` because the values of A and B will be used in B3's successor block, B4. Note that since C $\notin$ out[B3] C's value is *dead* and the assignment `C := 1` can be removed.

`out[B1]={A}∪{A,B}={A,B}` since if we take the left branch (through B2) A will be used further on, and if we take the right branch (through B3) both A and B will have a future use.

`in[B1]={B}` since B's value is used but not defined in B.

Classifying Data-Flow Problems

Classification — Forward vs. Backward Flow

- Data-flow problems can be classified according to the direction of flow:

  Forward-flow problems: Data flows from the initial block to the end block.
  - Out-sets are computed from In-sets within basic blocks,
  - In-sets are computed from Out-sets across basic blocks.

  Backward-flow problems: Data flows from the end block to the initial block.
  - In-sets are computed from Out-sets within basic blocks,
  - Out-sets are computed from In-sets across basic blocks.

Forward vs. Backward Flow — Equations

	Forward-Flow	Backward-Flow
Any Path	$\mathtt{o}_B = \mathtt{g}_B \cup (\mathtt{i}_B - \mathtt{k}_B)$ $\mathtt{i}_B = \bigcup_{b \in P(B)} \mathtt{o}_b$	$\mathtt{i}_B = \mathtt{g}_B \cup (\mathtt{o}_B - \mathtt{k}_B)$ $\mathtt{o}_B = \bigcup_{b \in S(B)} \mathtt{i}_b$
All Paths	$\mathtt{o}_B = \mathtt{g}_B \cup (\mathtt{i}_B - \mathtt{k}_B)$ $\mathtt{i}_B = \bigcap_{b \in P(B)} \mathtt{o}_b$	$\mathtt{i}_B = \mathtt{g}_B \cup (\mathtt{o}_B - \mathtt{k}_B)$ $\mathtt{o}_B = \bigcap_{b \in S(B)} \mathtt{i}_b$

- $P(B)$ = Predecessors of B, $S(B)$ = Successors of B.
- $\mathtt{i}_B = \mathtt{in}_B$, $\mathtt{o}_B = \mathtt{out}_B$, $\mathtt{g}_B = \mathtt{gen}_B$, $\mathtt{k}_B = \mathtt{kill}_B$, $\mathtt{o}_b = \mathtt{out}_b$, $\mathtt{i}_b = \mathtt{in}_b$.

Classification — Any- vs. All-Path

- We classify data-flow problems by the way they combine incoming information:

  Any-path problems: All values coming in to a block are valid. Use $\bigcup$.

  All-path problems: Only values coming in to a block through *every* path are valid. Use $\bigcap$.

	Forward-Flow	Backward-Flow
Any Path	Reaching Definitions Uninitialized Variables	Live Variables Du-chains
All Paths	Available Expressions Copy Propagation	Very Busy Expressions

Summary

Summary I

- Read the Dragon Book: 608–611, 614–615, 618, 620–622, 624–633.
- With B blocks & bit-vectors of length V, iterative data-flow analysis is $\mathcal{O}(B^2 \times V)$ in the worst case.

Homework

Homework I

- Show each step of the iterative reaching definitions algorithm applied to the procedure body below:

```
K := 1; I := 2;
REPEAT
   IF I = 4 THEN
      A := K + 1;
   ELSE
      A := K + 2;
      I := I + A;
   ENDIF;
UNTIL I <= 10;
K := K + A;
```

Exam Problems

Exam Problem I (a) [07.430 '95]

- An expression *E* is *very busy* if – regardless of which path we take through the flow graph – *E*'s value will be used before it is killed. Example (A+3 is *very busy*):

```
(1)   BEGIN
(2)      IF expr THEN
(3)         V := A + 3;
(4)         R := K + 3;
(5)      ELSE
(6)         Z := A + 3;
(7)         K := 5;
(8)         L := K + 3;
(9)      END;
(10)  END
```

Exam Problem I (b) [07.430 '95]

Data-Flow Equations:

- The data-flow equations for computing very busy expressions are:

$$\texttt{in}[B] = \texttt{used}[B] \cup (\texttt{out}[B] - \texttt{killed}[B])$$

$$\texttt{out}[B] = \bigcap_{\substack{\texttt{successors} \\ S \text{ of } B}} \texttt{in}[S]$$

Problems:

1. Give an iterative pseudo-code routine for computing `in` and `out`.
2. Is *very-busy expressions* a forward-flow or a backward-flow problem?
3. Show the workings of the algorithm on the procedure body in the next slide:

Exam Problem I (c) [07.430 '95]

```
BEGIN
   X := 5;
   Y := 10;
   IF e1 THEN
      IF e2 THEN
         A := X * Y;
      ELSE
         B := 3;
         V := X * Y;
         X := 1;
      END;
   ELSE
      Y := 2;
      A := X * Y;
   END
END
```