Research Statement

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1. Introduction

My research primarily focuses on the design and analysis of algorithms pertaining to the visualization of topological as well as geometric graphs. One key aesthetic is obtaining layouts with few or no crossings. Although standard planarity has been studied extensively, imposing additional restrictions such as drawing multiple planar layouts simultaneously, or finding a hierarchical layout with nodes arranged in horizontal levels, leads to a vast array of new possibilities.

Planar graphs have two very nice properties. First, they are closed under minors. Second, if a drawing exists with curves, then one exists with straight lines. One aspect of my research is finding the corresponding parallels with the alternate forms of planarity. Determining the underlying combinatorial and graph theoretic structures is vital to uncovering parallels when they exist.

2. Unlabeled Level Planarity

Hierarchical layouts partition the nodes of a given graph into levels in which nodes of the same level are drawn along the same horizontal line. Additionally, edges are drawn in a monotonic fashion such that no edge intersects any horizontal line more than once. Natural examples include scientific taxonomies, scheduling diagrams, and organizational charts where nodes are partitioned by classification, time, and rank, respectively. In these examples, levels often contain many nodes. By restricting each level to have only one vertex and requiring for the graph to be planar regardless of the ordering of the nodes among levels, I have uncovered a new type of planarity, which I call *unlabeled level planarity* [4, 5, 9].

Graphs with this type of planarity have several desirable properties. First, they have a well defined set of excluded minors. The graphs without these minors are also well defined, such as the set of all caterpillars, a special class of trees. Relatively few sets of graphs of topological interest are as well characterized. Second, they admit compact drawings taking linear time and space. The same graph can be drawn in a similar manner regardless of how the nodes are assigned to levels. This allows for dynamic visualization. Third, they can be recognized in linear time with a certificate proving or disproving planarity. For movies and demos of an application that has implemented all of the drawing and recognition algorithms for unlabeled level planar (ULP) trees, please visit the website `http://ulp.cs.arizona.edu`.

This type of planarity also has theoretical consequences. One is in determining whether a graph has a planar layout when drawn in levels. Patterns have been used to describe when this is possible. The set of patterns given by Healy *et al*. [11] were thought to be complete. However, some of the excluded minors for unlabeled level planarity did not match any of these patterns, and with the help of ULP trees we were able to find additional patterns [10]. More patterns than those given so far are known to exist, even for trees. Finding all remaining patterns, first for trees, and then for graphs with cycles, are both open problems.

Future work in this area also includes extending all these results to radial layouts, where levels are drawn on concentric circles and edges proceed in a radially monotonic fashion, and cyclic layouts, where levels are drawn on rays emanating from the origin and edges proceed strictly clockwise or counterclockwise.

3. Simultaneous Embedding

A generalization of planarity arises when drawing multiple graphs simultaneously. Here each graph is drawn on the same set of nodes in order to preserve the mental map between the different graphs. Moving a node to remove crossings in one graph may introduce crossings in another. The graphs share a *simultaneous embedding* if each is planar where crossings between edges of different graphs are not considered. Additionally, for the layout to be readable, edges between the same pair of nodes share the same curve when drawn.

Unlike standard planarity, topological graphs, where edge bends are allowed, and geometric graphs, where edges are straight, have different properties when drawn simultaneously. It is NP-hard to decide whether two geometric graphs, whereas it is NP-complete to decide whether three topological graphs share a simultaneous embedding. However, the complexity of deciding whether two topological graphs have a simultaneous embedding remains open. For topological graphs, a planar graph and a tree can always be embedded simultaneously, while two outerplanar graphs cannot. For geometric graphs, the pairs are more restrictive. Pairs of caterpillars can always be done, while pairs of trees cannot. However, it is unknown whether a path and a tree can always be done geometrically.

For geometric graphs, I have worked on finding techniques to draw a path and an outerplanar graph simultaneously where the path is drawn either with circular arcs or with two bends per edge [2, 3]. Future work includes checking whether this is possible with only one or no bends per edge. Colored simultaneous embedding is a generalization in which the vertices of the same color are mapped to points of the same corresponding color [1]. The question then becomes to find a colored pointset, preferrably universal, in which any graph of a given class, such as paths, can be draw on the colored pointset.

For topological graphs, I have found a set of forbidden pairs to characterize which pairs of graphs permit a simultaneous embedding [8]. This determines which graphs always share a simultaneous embedding with planar graphs. This allowed me to characterize which biconnected outerplanar graphs always have a simultaneous embedding with other outerplanar graphs, leading to polynomial-time decision and drawing algorithms. As an alternative approach, I have used SPQR-trees to decide when a pseudoforest (a graph with at most one cycle) has a simultaneous embedding with any planar graph [7]. For future work, I hope to use these results to find a polynomial-time decision algorithm for testing whether a pair of topological graphs have a simultaneous embedding. This would close the open question of whether its NP-hard or not to decide this problem.

Finally, I have participated in the construction of a tool that allows a user to dynamically manipulate multiple graphs simultaneously that supports all the variants of simultaneous embedding detailed above [6]. This tool is available for download at the website `http://graphset.cs.arizona.edu`.

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