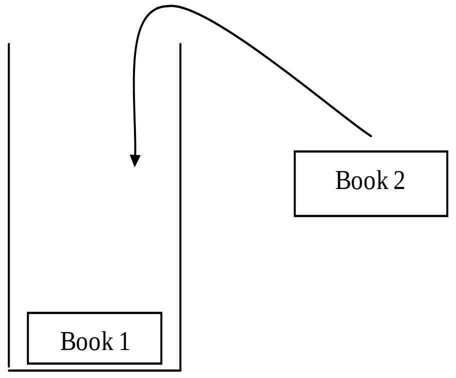
Chapter 18

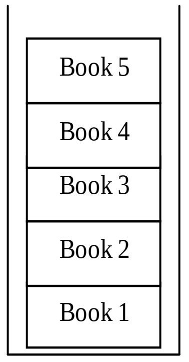
Stacks and Queues

18.1 Stacks

The stack abstract data type allows access to only one element—the one most recently added. This location is referred to as the top of the stack.

Consider how a stack of books might be placed into and removed from a cardboard box, assuming you can only move one book at a time. The most readily available book is at the top of the stack. For example, if you add two books—Book 1 and then Book 2—into the box, the most recently added book (Book 2) will be at the top of the stack. If you continue to stack books on top of one another until there are five, Book 5 will be at the top of the stack. To get to the least recently added book (Book 1), you first remove the topmost four books (Book 5, Book 4, Book 3, Book 2) — one at a time. Then the top of the stack would be Book 1 again.

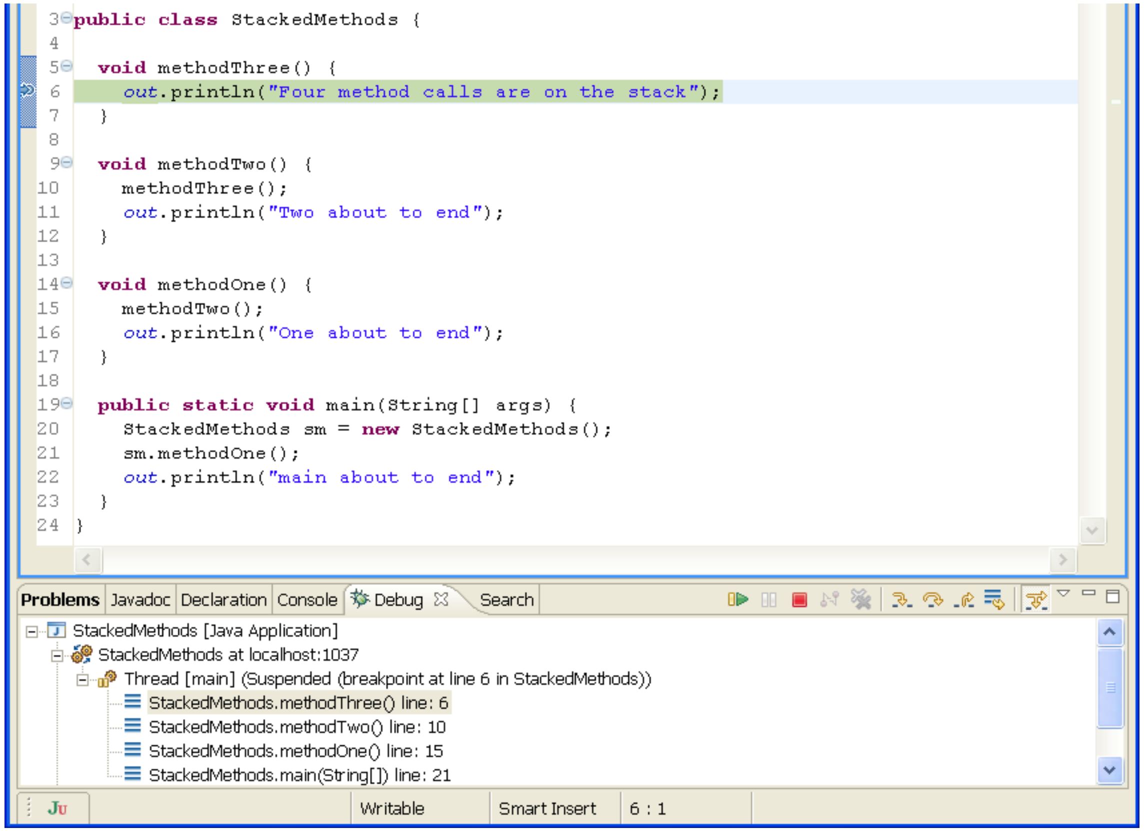




Stack elements are added and removed in a last in first out (LIFO) manner. The most recent element added to the collection will be the first element to be removed from the collection. Sometimes, the only data that is readily needed is the most recently accessed one. The other elements, if needed later, will be in the reverse order of when they were pushed. Many real world examples of a stack exist. In a physical sense, there are stacks of books, stacks of cafeteria trays, and stacks of paper in a printer's paper tray. The sheet of paper on the top is the one that will get used next by the printer.

For example, a stack maintains the order of method calls in a program. If `main` calls `function1`, that method calls `function2`, which in turn calls `function3`. Where does the program control go to when `function3` is finished? After `function3` completes, it is removed from the stack as the most recently added method. Control then returns to the method that is at the new top of the stack — `function2`.

Here is a view of the stack of function calls shown in a thread named `main`. This environment (Eclipse) shows the first method (`main`) at the bottom of the stack. `main` will also be the last method popped as the program finishes — the *first* method called is the *last* one to execute. At all other times, the method on the top of the stack is executing. When a method finishes, it can be removed and the method that called it will be the next one to be removed form the stack of method calls.



```java
public class StackedMethods {

  void methodThree() {
    out.println("Four method calls are on the stack");
  }

  void methodTwo() {
    methodThree();
    out.println("Two about to end");
  }

  void methodOne() {
    methodTwo();
    out.println("One about to end");
  }

  public static void main(String[] args) {
    StackedMethods sm = new StackedMethods();
    sm.methodOne();
    out.println("main about to end");
  }
}
```

```
StackedMethods [Java Application]
  StackedMethods at localhost:1037
    Thread [main] (Suspended (breakpoint at line 6 in StackedMethods))
      StackedMethods.methodThree() line: 6
      StackedMethods.methodTwo() line: 10
      StackedMethods.methodOne() line: 15
      StackedMethods.main(String[]) line: 21
```

The program output indicates the last in first out (or first in last out) nature of stacks:

```
Four method calls are on the stack
Two about to end
One about to end
main about to end
```

Another computer-based example of a stack occurs when a compiler checks the syntax of a program. For example, some compilers first check to make sure that `[ ]`, `{ }`, and ( ) are balanced properly. Thus, in a Java `class`, the final `}` should match the opening `{`. Some compilers do this type of symbol balance checking first (before other syntax is checked) because incorrect matching could otherwise lead to numerous error messages that are not really errors. A stack is a natural data structure that allows the compiler to match up such opening and closing symbols (an algorithm will be discussed in detail later).

A Stack Interface to capture the ADT

Here are the operations usually associated with a stack. (As shown later, others may exist):

• `push`	place a new element at the "top" of the stack
• `pop`	remove the top element and return a reference to the top element
• `isEmpty`	return `true` if there are no elements on the stack
• `peek`	return a reference to the element at the top of the stack

Programmers will sometimes add operations and/or use different names. For example, in the past, Sun programmers working on Java collection classes have used the name `empty` rather than `isEmpty`. Also, some programmers write their stack class with a `pop` method that does not return a reference to the element at the top of the stack. Our `pop` method will modify and access the state of stack during the same message.

Again, a Java interface helps specify Stack as an abstract data type. For the discussion of how a stack behaves, consider that `LinkedStack` (a collection class) implements the `OurStack` interface, which is an ADT specification in Java.

```
import java.util.EmptyStackException;

public interface OurStack<E> {
/** Check if the stack is empty to help avoid popping an empty stack.
  * @returns true if there are zero elements in this stack.
  */
  public boolean isEmpty();

/** Put element on "top" of this Stack object.
  * @param The new element to be placed at the top of this stack.
  */
  public void push(E element);

/** Return reference to the element at the top of this stack.
  * @returns A reference to the top element.
```

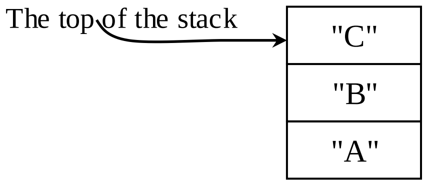
```
   * @throws EmptyStackException if the stack is empty.
   */
  public E peek() throws EmptyStackException;

/** Remove element at top of stack and return a reference to it.
   * @returns A reference to the most recently pushed element.
   * @throws EmptyStackException if the stack is empty.
   */
  public E pop() throws EmptyStackException;
}
```

You might need a stack of integers, or a stack of string values, or a stack of some new class of `Token` objects (pieces of source code). One solution would be to write and test a different stack class for each new class of object or primitive value that you want to store. This is a good reason for developing an alternate solution— a generic stack.

The interface to be implemented specifies the operations for a stack class. It represents the *abstract* specification. There is no particular data storage mentioned and there is no code in the methods. The type parameter `<E>` and return types `E` indicate that the objects of the implementing class will store any type of element. For example, `push` takes an `E` parameter while `peek` and `pop` return an `E` reference.

The following code demonstrates the behavior of the stack class assuming it is implemented by a class named LinkedStack.

```
// Construct an empty stack that can store any type of element
OurStack stackOfStrings<String> = new LinkedStack<String>();

// Add three string values to the stack
stackOfStrings.push("A");
stackOfStrings.push("B");
stackOfStrings.push("C");

// Show each element before each element is
while (! stackOfStrings.isEmpty()) {
  // Print the value of the element at the
  System.out.print(stackOfStrings.pop() + "
}
```



Output

```
C B A
```

Self-Check

18-1 Write the output generated by the following code:

```
OurStack<String> aStack = new LinkedStack<String> ();
```

```
        aStack.push("x");
        aStack.push("y");
        aStack.push("z");
        while (! aStack.isEmpty()) {
          out.println(aStack.pop());
        }
```

18-2 Write the output generated by the following code:

```
        OurStack<Character> opStack = new OurLinkedStack<Character>();
        out.println(opStack.isEmpty());
        opStack.push('>');
        opStack.push('+');
        opStack.push('<');
        out.print(opStack.peek());
        out.print(opStack.peek());  // careful
        out.print(opStack.peek());
```

18-3 Write the output generated by the following code:

```
        OurStack<Integer> aStack = new OurLinkedStack<Integer>();
        aStack.push(3);
        aStack.push(2);
        aStack.push(1);
        out.println(aStack.isEmpty());
        out.println(aStack.peek());
        aStack.pop();
        out.println(aStack.peek());
        aStack.pop();
        out.println(aStack.peek());
        aStack.pop();
        out.println(aStack.isEmpty());
```

18.2 Stack Application: Balanced Symbols

Some compilers perform symbol balance checking before checking for other syntax errors. For example, consider the following code and the compile time error message generated by a particular Java compiler (your compiler may vary).

```
public class BalancingErrors
  public static void main(String[] args) {
    int x = p;
```

```
BalancingErrors.java:1: '{' expected
public class BalancingErrors
                            ^
```

```
    int y = 4;
    in z = x + y;
    System.out.println("Value of z = " + z);
  }
}
```

Notice that the compiler did not report other errors, one of which is on line 3. There should have been an error message indicating `p` is an unknown symbol. Another compile time error is on line 5 where `z` is incorrectly declared as an `in` not `int`. If you fix the first error by adding the left curly brace on a new line 1 you will see these other two errors.

```
public class BalancingErrors  {  // <- add an opening curly brace
  public static void main(String[] args) {
    int x = p;
    int y = 4;
    in z = x + y;
    System.out.println("Value of z = " + z);
  }
}

BalancingErrors.java:3: cannot resolve symbol
symbol  : variable p
location: class BalancingErrors
    int x = p;
            ^
BalancingErrors.java:5: cannot resolve symbol
symbol  : class in
location: class BalancingErrors
    in z = x + y;
    ^
2 errors
```

This behavior could be due to a compiler that first checks for balanced { and } symbols before looking for other syntax errors.

Now consider how a compiler might use a stack to check for balanced symbols such as ( ) , `{  }`, and `[  ]`. As it reads the Java source code, it will only consider opening symbols: `(  {  [`, and closing symbols:) `}  ]`. If an opening symbol is found in the input file, it is pushed onto a stack. When a closing symbol is read, it is compared to the opener on the top of the stack. If the symbols match, the stack gets popped. If they do not match, the compiler reports an error to the programmer. Now imagine processing these tokens, which represent only the openers and closers in a short Java program: `{{([])}}`. As the first four symbols are read — all openers — they get pushed onto the stack.

Java source code starts as: `{{([])}}`

```
[
(
{
{
```

push the first four opening symbols with **[** *at the top. Still need to read* **]) } }**

The next symbol read is a closer: `"]"`. The `"["` would be popped from the top of the stack and compared to `"]"`. Since the closer matches the opening symbol, no error would be reported. The stack would now look like this with no error reported:

```
(
{
{
```

pop `[` *which matches* `]`*. There is no error. Still need to read* **) } }**

The closing parenthesis `")"` is read next. The stack gets popped again. Since the symbol at the top of the stack `"("` matches the closer `")"`, no error needs to be reported. The stack would now have the two opening curly braces.

```
{
{
```

pop `(` *which matches). There is no error. Still need to read* **} }**

The two remaining closing curly braces would cause the two matching openers to be popped with no errors. It is the last-in-first-out nature of stacks that allows the first pushed opener `"{"` to be associated with the last closing symbol `"}"` that is read.

Now consider Java source code with only the symbols `(]`. The opener `"("` is pushed. But when the closer `"]"` is encountered, the popped symbol `"("` does not match `"]"` and an error could be reported. Here are some other times when the use of a stack could be used to help detect unbalanced symbols:

1. If a closer is found and the stack is empty. For example, when the symbols are `{}}`. The opening `{` is pushed and the closer `"}"` is found to be correct. However when the second `}` is encountered, the stack is empty. There is an error when `}` is discovered to be an extra closer (or perhaps `{` is missing).
2. If all source code is read and the stack is not empty, an error should be reported. This would happen with Java source code of `{{([])}`. In this case, there is a missing right curly brace. Most symbols are processed without error. At the end, the stack *should* be empty. Since the stack is *not* empty, an error should be reported to indicate a missing closer.

This algorithm summarzies the previous actions.

```
1. Make an empty stack named s
2. Read symbols until end of file
      if it's an opening symbol, push it
      if it is a closing symbol && s.empty
          report error
     otherwise
          pop the stack
          if symbol is not a closer for pop's value, report error
 3. At end of file, if !s.empty, report error
```

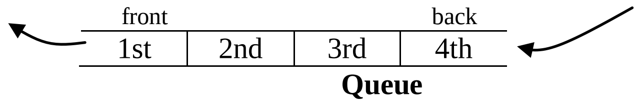
Self-Check

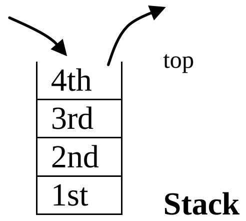
18-4 Write the errors generated when the algorithm above processes the following input file:

```
public class Test2 {
   public static void main(String[] args) {
     System.out.println();
  )) {
```

18.3 FIFO Queues

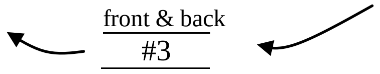
A first-in, first-out (FIFO) **queue** — pronounced "Q" — models a waiting line. Whereas stacks add and remove elements at one location — the `top` — `queues` add and remove elements at different locations. New elements are added at the back of the queue. Elements are removed from the front of the queue.



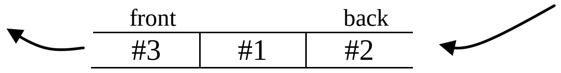


Whereas stacks mimic LIFO behavior, queues mimic a first in first out (FIFO) behavior. So, for example, the queue data structure models a waiting line such as people waiting for a ride at an amusement park. The person at the front of the line will be the first person to ride. The most recently added person must wait for all the people in front of them to get on the ride. With a FIFO queue, the person waiting longest in line is served before all the others who have waited less time[1].

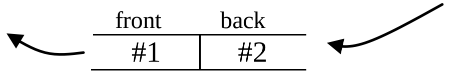
Another example of queue behavior can be found when several documents need to be printed at a shared printer. Consider three students, on the same network, trying to print one document each. Who gets their document printed first? If a FIFO queue is being used to store incoming print requests, the student whose request reached the print queue first will get printed ahead of the others. Now assume that the printer is busy and the print queue gets a print request from student #3 while a ducument is printing. The print queue would look something like this:



In this case the queue’s front element is also at the back end of the queue. The queue contains one element. Now add another request from student #1, followed by another request from student #2 for printing, and the print queue would look like this:



Student #1 and student #2 requests were added to the back of queue. The print requests are stored in the order in which they arrived. As the printer prints documents, the document will be removed from the front. Once the printer has printed the current document, the document for student #3 will then be removed. Then the state of the queue will now look like this:



A Queue Interface — Specifying the methods

There is no universally agreed upon set of operations; however the following is a reasonable set of operations for a FIFO Queue ADT.

[1] Note: A *priority queue* has different behavior where elements with a higher priority would be removed first. For example, the emergency room patient with the most need is attended to next, not the patient who has been there the longest.

• isEmpty	Return true only when there are zero elements in the queue
• add	Add an element at the back of the queue
• peek	Return a reference to the element at the front of the queue
• remove	Return a reference to the element at the front and remove the element

This leads to the following interface for a queue that can store any class of object.

```java
public interface OurQueue<E> {

   /**
    * Find out if the queue is empty.
    * @returns true if there are zero elements in this queue.
    */
  public boolean isEmpty();

   /**
    * Add element to the "end" of this queue.
    * @param newEl element to be placed at the end of this queue.
    */
  public void add(E newEl);

   /**
    * Return a reference to the element at the front of this queue.
    * @returns A reference to the element at the front.
    * @throws NoSuchElementException if this queue is empty.
    */
  public E peek();

   /**
    * Return a reference to front element and remove it.
    * @returns A reference to the element at the front.
    * @throws NoSuchElementException if this queue is empty.
    */
  public E remove();
}
```

The following code demonstrates the behavior of the methods assuming OurLinkedQueue implements interface OurQueue:

```java
OurQueue<Integer> q = new OurLinkedQueue<Integer>();
q.add(6);
q.add(2);
```

```
q.add(4);
while (!q.isEmpty()) {
  System.out.println(q.peek());
  q.remove();
}
```

Output

```
6 2 4
```

Self-Check

18-5 Write the output generated by the following code.

```
OurQueue<String> stringQueue = new OurLinkedQueue<String>();
stringQueue.add("J");
stringQueue.add("a");
stringQueue.add("v");
stringQueue.add("a");
while (!stringQueue.isEmpty()) {
  System.out.print(stringQueue.remove());
}
```

18-6 Write the output generated by the following code until you understand what is going on.

```
OurQueue<String> stringQueue = new OurLinkedQueue<String>();
stringQueue.add("first");
stringQueue.add("second");
while (! stringQueue.isEmpty()) {
  System.out.println(stringQueue.peek());
}
```

18-7 Write code that displays a message to indicate if each integer in a queue named `intQueue` is even or odd. The queue must remain intact after you are done. The queue is initialized with random integers in the range of 0 through 99.

```
OurQueue<Integer> intQueue = new OurLinkedQueue<Integer>();
Random generator = new Random();
for(int j = 1; j <= 7; j++) {
  intQueue.add(generator.nextInt(100));
}
// Your solution goes here
```

Sample Output (output varies since random integers are added)

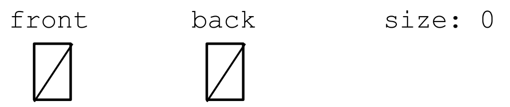
```
28 is even
72 is even
4 is even
37 is odd
94 is even
98 is even
33 is odd
```

18.4 Queue with a Linked Structure

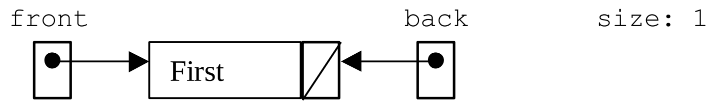
We will implement interface OurQueue with a class that uses a singly linked structure. There are several reasons to choose a linked structure over an array:

- It is easier to implement. (A programming project explains the trickier array-based implementation).
- The Big-O runtime of all algorithms is as efficient as if an array were used to store the elements. All algorithms can be O(1).
- An array-based queue would have `add` and `remove` methods, which will occasionally run O(n) rather than O(1). This occurs whenever the array capacity needs to be increased or decreased.
- It provides another good example of implementing a data structure using the linked structure introduced in the previous chapter.

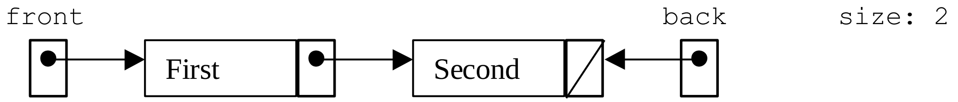
Elements are removed from the "front" of a queue. New elements are added at the back of the queue. Both "ends" of the queue are frequently accessed. Therefore, this implementation of `OurQueue` will use two external references. Only one external reference to the front is required. However, this would make for O(n) behavior during `add` messages, since a loop would need to sequence through all elements before reaching the end. With only a reference to the front, all elements must be visited to find the end of the list before one could be added. Therefore, an external reference named `back` will be maintained in addition to front. This will allow `add` to be O(1). An empty OurLinkedQueue will look like this:



After `q.add("First")`, a queue of size 1 will look like this:



After `q.add("Second")`, the queue of size 2 will look like this:



This test method shows the changing state of a queue that follows the above pictures of memory.

```
  @Test
  public void testAddAndPeek() {
    OurQueue<String> q = new OurLinkedQueue<String>();
    assertTrue(q.isEmpty()); // front == null
    q.add("first");
    assertEquals("first", q.peek());  // front.data is "first"
    assertFalse(q.isEmpty());

    q.add("second");  // Change back, not front
    // Front element should still be the same
    assertEquals("first", q.peek());
  }
```

The first element is accessible as `front.data`. A new element is added by storing a reference to the new node into `back.next` and adjusting `back` to reference the new node at the end.

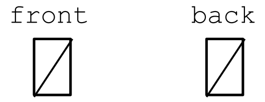
Here is the beginning of class OurLinkedQueue that once again uses a private inner `Node` class to store the data along with a link to the next element in the collection. There are two instance variables to maintain both ends of the queue.

```
public class OurLinkedQueue<E> implements OurQueue<E> {

  private class Node {
    private E data;
    private Node next;

    public Node() {
      data = null;
      next = null;
    }

    public Node(E elementReference, Node nextReference) {
      data = elementReference;
      next = nextReference;
    }
  } // end class Node
  // External references to maintain both ends of a Queue
  private Node front;
  private Node back;

  /**
   * Construct an empty queue (no elements) of size 0.
   */
  public OurLinkedQueue() {
    front = null;
    back = null;
  }

 /** Find out if the queue is empty.
  * @returns true if there are zero elements in this queue.
  */
  public boolean isEmpty()  {
    return front == null;
  }

  // More methods to be added . . .
}
```

This implementation recognizes an empty queue when `front` is `null`.

add

The `add` operation will first check for the special case of adding to an empty queue. The code to add to a non-empty queue is slightly different. If the queue is empty, the external references front and back are both `null`.



In the case of an empty queue, the single element added will be at front of the queue and also at the back of the queue. So, after building the new node, front and back should both refer to the same node. Here is a before and after picture made possible with the code shown.

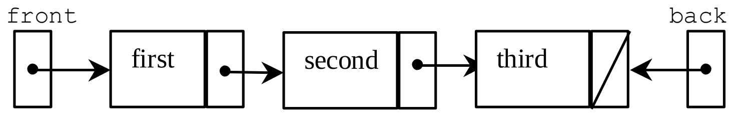
```
  // Build a node to be added at the end. A queue can
  // grow as long as the computer has enough memory.
  // With a linked structure, resizing is not necessary.
  if (this.isEmpty()) {
     front = new Node(element, null);
     back = front;
   }
```

When an add messages is sent to a queue that is not empty, the last node in the queue must be made to refer to the node with the new element. Although `front` must remain the same during `add` messages, `back` must be changed to refer the new element at the end.

```
  else {
    back.next = new Node(element);
    back = back.next;
  }
```

There are several viable variations of how algorithms could be implemented when a linked structure is used to store the collection of elements. The linked structure used here always maintains two external references for the `front` and `back` of the linked structure. This was done so `add` is O(1) rather than O(n). In summary, the following code will generate the linked structure shown below.

```
OurQueue<String> q = new OurLinkedQueue<String>();
q.add("first");
q.add("second");
q.add("third");
```



Self-Check

18-8 Draw a picture of what the memory would look like after this code has executed

```
     OurQueue<Double> q1 = new OurLinkedQueue<Double>();
     q1.add(5.6));
     q1.add(7.8));
```

18-9 Implement a `toString` method for `OurLinkedQueue` so this assertion would pass after the code in the previous self-check question:

```
    assertEquals("[a, b]", q2.toString());
```

peek

The `peek` method throws a `NoSuchElementException` if the queue is empty. Otherwise, `peek` returns a reference to the element stored in front.data.

```
  /**
   * Return a reference to the element at the front of this queue.
   * @returns A reference to the element at the front.
   * @throws NoSuchElementException if this queue is empty.
   */
  public E peek() {
    if (this.isEmpty())
      throw new java.util.NoSuchElementException();
    else
      return front.data;
  }
```

The next two test methods verify that `peek` returns the expected value and that it does not modify the queue.

```
  @Test
  public void testPeek() {
    OurQueue<String> q = new OurLinkedQueue<String>();
    q.add(new String("first"));
    assertEquals("first", q.peek());
    assertEquals("first", q.peek());

    OurQueue<Double> numbers = new OurLinkedQueue<Double>();
    numbers.add(1.2);
    assertEquals(1.2, numbers.peek(), 1e-14);
    assertEquals(1.2, numbers.peek(), 1e-14);
  }


  public void testIsEmptyAfterPeek() {
    OurQueue<String> q = new OurLinkedQueue<String>();
    q.add("first");
    assertFalse(q.isEmpty());
    assertEquals("first", q.peek());
  }
```

An attempt to peek at the element at the front of an empty queue results in a `java.util.NoSuchElementException`, as verified by this test:

```
  @Test(expected = NoSuchElementException.class)
  public void testPeekOnEmptyList() {
    OurQueue<String> q = new OurLinkedQueue<String>();
    q.peek();
  }
```

`remove`

The `remove` method will throw an exception if the queue is empty. Otherwise, `remove` returns a reference to the object at the front of the queue (the same element as `peek()` would). The `remove` method also removes the front element from the collection.

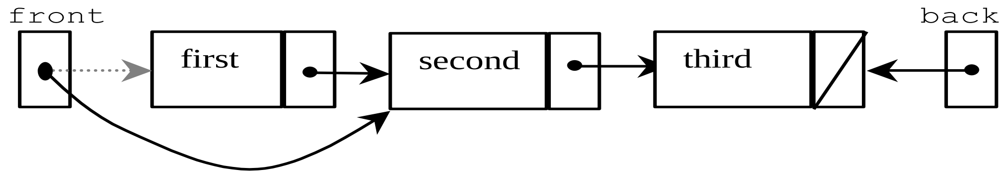
```
 @Test
  public void testRemove() {
    OurQueue<String> q = new OurLinkedQueue<String>();
    q.add("c");
    q.add("a");
    q.add("b");
    assertEquals("c", q.remove());
    assertEquals("a", q.remove());
    assertEquals("b", q.remove());
  }

  @Test(expected = NoSuchElementException.class)
  public void testRemoveThrowsAnException() {
    OurQueue<Integer> q = new OurLinkedQueue<Integer>();
    q.remove();
  }
```

Before the front node element is removed, a reference to the front element must be stored so it can be returned after removing it.

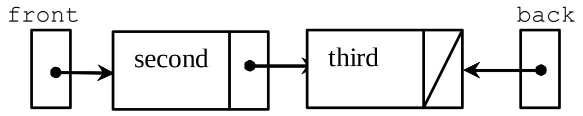
```
E frontElement = front.data;
```

`front`'s next field can be sent around the first element to eliminate it from the linked structure.

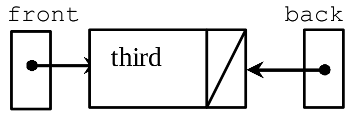
```
front = front.next;
```



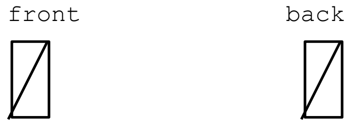
Now the method can return a reference to `firstElement`. The linked structure would now look like this.



Another remove makes the list look like this.



Another remove message will return "third". The `remove` method should set front to null so isEmpty() will still work. This will leave the linked structure like this with back referring to a node that is no longer considered to be part of the queue. In this case, back will also be set to null.



Self-Check

18-10 Complete method `remove` so it return a reference to the element at the front of this queue while removing the front element. If the queue is empty, `throw new NoSuchElementException()`.

```
  public E remove() {
```

Answers to Self-Checks

18-1
```
z
y
x
```

18-2
```
true
<<<
```

18-3
```
false
1
2
3
true
```

18-4
```
Check symbols in Test2.java
Abc.java:2 expecting ]
Abc.java:4 expecting }
Abc.java:4 expecting }
missing }
4 errors
```

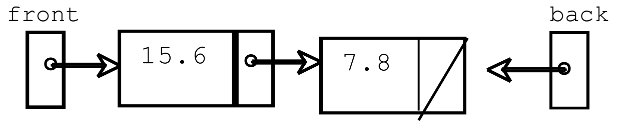
18-5
```
Java
```

18-6
```
first
first
first
  ...
```

```
first until someone terminates the program or the power goes out
```

18-7
```
int size = intQueue.size();
for (int j = 1; j <= size; j++) {
  int nextInt = intQueue.peek();
  if (nextInt % 2 != 0)
     System.out.println(nextInt + " is odd");
  else
     System.out.println(nextInt + " is even");
  intQueue.remove();
  intQueue.add(nextInt);
}
```

18-8



18-9
```
public String toString() {
   String result = "[";

   // Concatenate all but the last one (if size > 0)
   Node ref = front;
   while (ref != back) {
     result += ref.data + ", ";
     ref = ref.next;
   }

   // Last element does not have ", " after it
   if (ref != null)
     result += ref.data;

   result += "]";

   return result;
}
```

18-10
```
public E remove() throws NoSuchElementException {
   if (this.isEmpty())
     throw new NoSuchElementException();

   E frontElement = front.data;
   front = front.next;

   if (front == null)
     front = back = null;

   return frontElement;
}
```