Principles of Programming Languages

Lecture 04

Types and Polymorphism

Types

- What is a type?
  - An equivalence class of objects/values
    - Denotational view: a type is a set (of values): Pascal
      - `type weekday = (sun, mon, tue, wed, thu, fri, sat);`

        $$Weekday = \{sun, mon, tue, wed, thu, fri, sat\}$$
    - Constructive view: a type is the result of an expression consisting of primitive types operated upon by type constructors: Ada

```
type computer is record
    serial : array (1..10) of integer;
    age: integer;
end record;
```

    - Abstraction view: a type is an interface, providing a set operations on objects of the type; an abstract data type (ADT):
      - Pascal: `pred(), succ(), <, =, >`
      - Class declaration

Types (cont.)

- What has a type?
  - Literals `1.25 ‘abc’`
  - Variables **var** *x*: **integer**;
  - Expressions `x:int + y` ML type `int`
    - Induced by types of variables, literals, operators (casting ops included), and any implicit coercion (conversion) rules
    -
  - Objects `Stack<int> s;`
  - Functions `-fun area(r) = 3.141582818*r*r;`
    `val area = fn : real -> real;`
  - References `int& x;` `x:ref int;`
  - Pointers `int i = 3; int& r = i; int* p = &r;`

Type System

- Type definition rules
  - Declaration (naming): introduce new name & bind to scope
  - Definition (description):
    - Primitive: booleans, characters, integers, fixedpoint, floating point
    - Enumeration: Ada `type weekday is (sun, … ,sat);`
    - Subtype: `subtype weekend is weekday range sat..sun;`
    - Composite: `record, union, array, reference, list` (type "operators")
    - Function: C++: `int max(int a, int b){return a>b?a:b;}`
    - Derived: Ada: `type mass is new REAL;`
- Type equivalence rules
  - Name equivalence
    - Each definition a new type
    - Equivalent only if declared as same primitive or pre-defined type

    Ada distinct types: `a,b: array(1..10) of BOOLEAN;`
  - Declaration equivalence
    - Same declaration implies same type (example above is dec. equiv.)

Type System(cont.)

- Structural Equivalence: have same type-operator expression

- Type compatibility rules
  - Argument/parameter compatibility; assignment compatibility
  - Types might be different but compatible; rules differ widely
    - Ada : a subtype is compatible with a supertype & arrays of same size & base type are compatible
    - C: `short int s; unsigned long int l; … ; s = l;`
    - `C: void* p; int* q; … ; q = p;`
    - *Coercion:* implicit type conversion defined by language (≠ *cast*)
  - Type Checking: verifying a program adheres to type compatibility rules (e.g. `lint` a type checker for a weakly typed `C`)
    - *Strong* typing: prohibits an op when incompatibility exists
      - Ada strongly typed. Bliss untyped. ANSI C in middle
    - *Static* type checking: compile time (Ada, C++)
    - *Dynamic* type checking: late binding (Lisp, Scheme, Smalltalk)

Type System (cont.)

- Type Inference Rules
  - Rules for typing an expression given the types of its components
    - Type of **`x = y;`** is type of **`x`**
    - Type of **`b?a:b`** is the (common) type of **`a, b`** etc, etc
    - Ada: “con” & “cat” (both array[1..3] of char) returns array[1..6] of char
  - Subranges `x:INTEGER range 0..40; y:INTEGER range 10..20;` type of x + y ?
  - Can be complex, and involve coercion
    - Recall PL/I example with fixed bin and fixed dec operands
  - Some inferences impossible at compile time
  - Inference is a kind of “evaluation” of expressions having coarse values; types have their own arithmetic

Polymorphism

- A polymorphic subroutine is one that can accept arguments of different types for the same parameter
  - `max(x,y){ max = x>y?x:y }` *could* be reused for any type for which > is well-defined
- A polymorphic variable(parameter) is one that can refer to objects of multiple types. ML: `x : 'a`
- True (or "pure") polymorphism always implies *code reuse:* the *same* code is used for arguments of *different* types.
- What polymorphism is not:
  - *Not overloading*.
  - *Not generics.*
  - *Not coercion.*
  - All 4 aim at off-loading effort from programmer to translator, but in different ways

Polymorphism(cont.)

- *Overloading*
  - An *overloaded* name refers to several distinct objects in the same scope; the name's reference (denotation) is resolved by context. Unfortunately sometimes called "*ad hoc* polymorphism"(!)
  - C++

```
int j,k; float r,s;
int max(int x, int y){ return x<=y?y:x }
float max(float x, float y){ return y>x?y:x }
…
max(j,k); // uses int max
max(r,s); // uses float max
```

  - Even constants can be overloaded in Ada:

```
type weekday is (sun, mon, …);
type solar is (sun, merc, venus, …);
planet: solar; day: weekday;
day := sun;  planet := sun;    -- compatible
day := planet;   -- type error
```

Polymorphism(cont.)

- *Generic* subroutines
  - A generic subroutine is a syntactic *template* containing a type parameter that can be used to generate different code for each type instantiated
  - Ada

```
generic
  type T is private;
  with function "<="(x, y : T) return Boolean;
function max(x,y : T) return T is
begin if x <= y then return y;
      else return x;
      end if;
end min;
function bool_max is new max(BOOLEAN,implies);
function int_max is new max(INTEGER,"<=");
```

Polymorphism(cont.)

- *Coerced* subroutine arguments
  - A coercion is a built-in compiler conversion from one type to another
  - Fortran

```
function rmax(x,y)
real x
real y
rmax=x
if (y .GT. x) rmax=y
return
end
```

  - In `k=rmax(i,j)` causes args to be coerced to floating point & return value truncated to integer
  - Although *same code* is used for both arg types, this is not true polymorphism

Kinds of Polymorphism

- *Pure polymorphism:* a single subroutine can be applied to arguments of a variety of types
  - *Parametric* polymorphism: the type value is passed explicitly as an argument. There is a type called **type** in CLU:

```
sorted_bag = cluster[t:type] is create, insert, …
  where t has lt,eq: proctype(t,t) returns (bool);
…
wordbag := sorted_bag[string];    -- create cluster
wb: wordbag := wordbag$create(); -- instance
…
wordbag$insert(wb, word);        -- mutate instance
```

Kinds of Polymorphism(cont.)

- *Type variable* polymorphism: a type signature with type variables is derived for each subroutine that is as general as possible (unification). An applied subroutine has its type variables instantiated with particular types.

```
- fun length(nil) = 0
=   | length(a :: y) = 1 + length(y);
val length = fn : 'a list -> int

- val a = ["a", "b", "c"];
val a = ["a","b","c"] : string list

- length(a);
val it = 3 : int

- val b = [1,3,5,7,21,789];
val b = [1,3,5,7,21,789] : int list

- length(b);
val it = 6 : int
```

Kinds of Polymorphism(cont.)

```
- val d = [35,3.14];
std_in:0.0-0.0 Error: operator and operand don't agree (tycon
   mismatch)
  operator domain: int * int list  operand: int * real list in
  expression: 35 :: 3.14 :: nil

- val e = [3.14, 2.71828, 1.414];
val e = [3.14,2.71828,1.414] : real list
- length(e);
val it = 3 : int
```

Kinds of Polymorphism(cont.)

- *Late Binding* polymorphism: deferral of type checks to run-time allows polymorphic code to be written once and used with different types

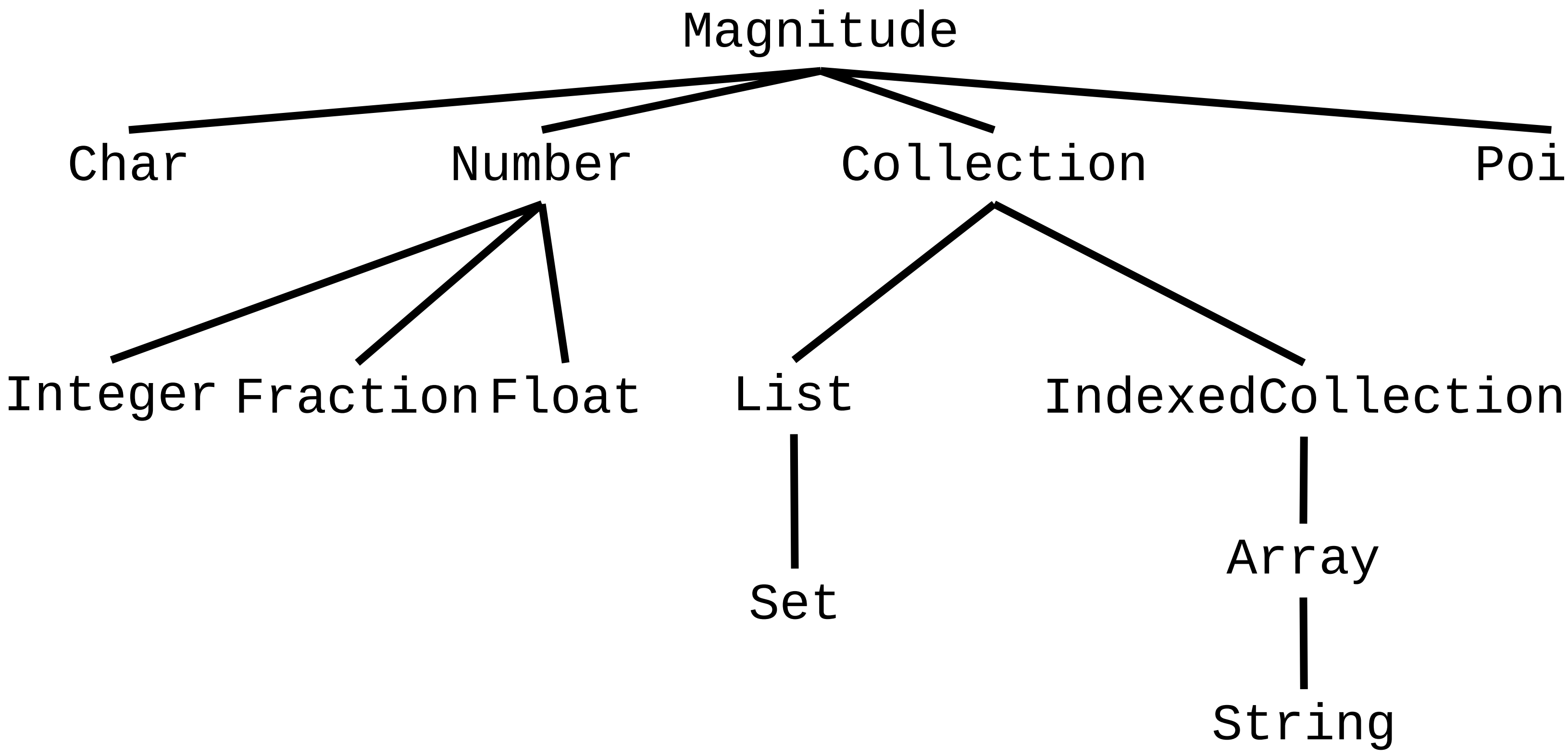
```
caslon> cat length.scm
;;;; length - return length of a list
(define length
  (lambda (x)
    (if (null? x)
        0
        (1+ (length (cdr x)))
)))
caslon> scheme
1 ]=> (load "length.scm")

1 ]=> (define a (list 2 7 1 8 28 1 8))
A
1 ]=> (length a)
7
1 ]=> (define a (list "foo" "baz" "snafu"))
A
1 ]=> (length a)
3
```

Kinds of Polymorphism(cont.)

- *Inheritance* polymorphism: one class method executed on objects of distinct subclasses; common code is "inherited".

Ex: in Little Smalltalk the subclasses of Magnitude are



Kinds of Polymorphism(cont.)

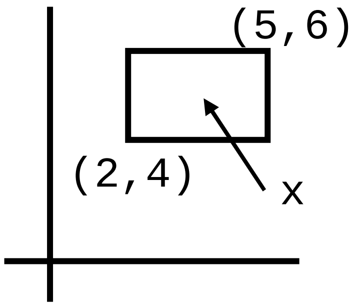
^name = value of name

- An implementation of class Magnitude

```
Class: Magnitude
Instance variables:
Instance methods:
<n    ^self   implementedBySubclass
=n    ^self   implementedBySubclass
<=n   ^(self < n) or: (self = n)
>n    ^ (self <= n) not
>=n   ^(self < n) not
between: min and: max
  ^ (min <= self) and: (self <= max)
max: n
  (self > n)
      ifTrue:  [^self]
      ifFalse: [^n]
min: n
  (self < n)
      ifTrue:  [^self]
      ifFalse: [^n]
```

Kinds of Polymorphism(cont.)

- Invocation with different classes (types)
  - Char: `x between: $a and: $z`
    - If `x` is a `Char`, method is not found at `Char.` Search proceeds up to superclass `Magnitude. ^ ($a <= x) & (x <= $z)` invoked. First `<=x` sent to `$a` of class `Char,` where method `<=` is not found, …, in `Magnitude` invoke `^ ($a < x) or ($a = x).` This sends message `<x` to `$a` and this method is found in class `Char.` Suppose `x` is actually `$b`. Eventually the ob `true` is sent message `or:false`, resulting in value `true.` So result of `($a <= x)` is now effectively determined at `$a`, returning a `true` ob. … Eventually, by similar process this `true` will be sent `and:true`, so it returns itself
  - String: `‘carbon’ between: ‘carbolic’ and: ‘carbonate’`
  - Point: `x between: 2@4 and: 5@6`



- All use same code!

ML: Strong Typing & Polymorphism

```
lec> sml
Standard ML of New Jersey, Version 110.0.6, October 31, 1999
val use = fn : string -> unit
- fun succ n = n+1;
val succ = fn : int -> int
- succ "zero";
stdIn:7.1-7.12 Error: operator and operand don't agree [tycon mismatch]
  operator domain: int operand: string in expression: succ "zero"
- succ 3;
val it = 4 : int
- fun add(x,y) = x + y;
val add = fn : int * int -> int
- add 3 5;
stdIn:9.1-9.8 Error: operator and operand don't agree [literal]
  operator domain: int * int operand: int in expression: add 3
- add (3,5);
val it = 8 : int
- fun I x = x;
GC #0.0.0.0.1.5:   (0 ms)
val I = fn : 'a -> 'a
```

ML (cont.)

```
- fun self = ( x x);
stdIn:11.14-11.20 Error: operator is not a function [circularity]
  operator: 'Z in expression: x x
- fun apply f x = (f x);
val apply = fn : ('a -> 'b) -> 'a -> 'b
- apply succ 7;
val it = 8 : int
- add 3;
stdIn:13.1-13.6 Error: operator and operand don't agree [literal]
  operator domain: int * int operand: int in expression: add 3
- fun plus x y = x + y;
val plus = fn : int -> int -> int
- plus 3;
val it = fn : int -> int
- plus 3 5;
val it = 8 : int
- val add3 = plus 3;
val add3 = fn : int -> int
- add3 5;
val it = 8 : int
- fun K x y = x;
val K = fn : 'a -> 'b -> 'a
```

ML (cont.)

```
- K I;
stdIn:19.1-19.4 Warning: type vars not generalized because of
   value restriction are instantiated to dummy types (X1,X2,...)
val it = fn : ?.X1 -> ?.X2 -> ?.X2
- K I 3;
stdIn:20.1-20.6 Warning: type vars not generalized because of
   value restriction are instantiated to dummy types (X1,X2,...)
val it = fn : ?.X1 -> ?.X1
- K I 3 24;
val it = 24 : int
- K I "foo" 24;
val it = 24 : int
- K succ;
stdIn:23.1-23.7 Warning: type vars not generalized because of
   value restriction are instantiated to dummy types (X1,X2,...)
val it = fn : ?.X1 -> int -> int
- K succ 3;
val it = fn : int -> int
- K succ 3 15;
val it = 16 : int
- ^D
```

ML: Polymorphic Reference Types

```
- (* can have refs to variable types *)
- val a = ref 7;
val a = ref 7 : int ref
- val b = ref 11;
val b = ref 11 : int ref
- !a;
val it = 7 : int
- !b;
val it = 11 : int
- fun swap (x, y) =
=   let val temp = !x
=   in x := !y; y := temp
=   end;
val swap = fn : 'a ref * 'a ref -> unit
- swap(a,b);
val it = () : unit
- !a;
val it = 11 : int
- !b;
val it = 7 : int
```

ML: Reference Types (cont.)

```
- val c = ref true;
val c = ref true : bool ref
- val d = ref false;
val d = ref false : bool ref
- swap(c,d);
val it = () : unit
- !c;
val it = false : bool
- !d;
val it = true : bool
- swap(a,c);
std_in:29.1-29.9 Error: operator and operand don't agree (tycon
   mismatch)
  operator domain: int ref * int ref operand: int ref * bool ref in
   expression: swap (a,c)
```

ML: Static Typing

```
opu> scheme
1 ]=> ;;;;;; a function acceptable to Scheme but not type-correct in
   ML
      (define applyto
        (lambda (f) (cons (f 3) (f "hi")) ))
APPLYTO
1 ]=> (applyto (lambda (x) x)  )
(3 . "hi")
1 ]=> (applyto (lambda (x) (cons 'glurg x)))
((GLURG . 3) GLURG . "hi")    ;;; ((GLURG . 3) (GLURG . "hi"))
opu> sml
- (* ML type-inference algorithm unwilling to accept APPLYTO *)
- val applyto = fn f => ( f(3), f("hi") );
std_in:11.23-11.39 Error: operator and operand don't agree (tycon
   mismatch)
  operator domain: int operand: string in expression: f ("hi")
- (* Below there are two insances of I x = x that take distinct types.
=   Why??  *)
- let fun I x = x in   ( I(3), I("hi") )  end;
val it = (3,"hi") : int * string
```

ML & λ-Calculus

```
lec> script skk
Script started on Tue Feb 19 09:01:20 200
lec> sml
Standard ML of New Jersey, Version 110.0.6, October 31, 1999
val use = fn : string -> unit
- fun I x =  x;
val I = fn : 'a -> 'a
- fun add x y = x + y;
val add = fn : int -> int -> int
- fun add1 z = add 1 z;
val add1 = fn : int -> int
- add1 10;
GC #0.0.0.0.1.4:   (0 ms)
val it = 11 : int
- fun twice f x = f (f x);
val twice = fn : ('a -> 'a) -> 'a -> 'a
- twice add1 5;
val it = 7 : int
- fun mul2 x = 2*x;
val mul2 = fn : int -> int
- mul2 10;
val it = 20 : int
```

ML & λ-Calculus (cont)

```
- fun S x y z = x z (y z);
val S = fn : ('a -> 'b -> 'c) -> ('a -> 'b) -> 'a -> 'c
- S add mul2 5;
val it = 15 : int
- S add I 5;
val it = 10 : int
- fun K x y = x;
val K = fn : 'a -> 'b -> 'a
- fun T z = S K z;
val T = fn : ('a -> 'b) -> 'a -> 'a
- fun V w = T K w;
val V = fn : 'a -> 'a
- V 3;
val it = 3 : int
- V 10;
val it = 10 : int
- V 20;
val it = 20 : int
- S K K 10;
val it = 10 : int
- S K K 21;
val it = 21 : int
```

ML & λ-Calculus (cont)

```
- S I I;
stdIn:26.1-26.6 Error: operator and operand don't agree [circularity]
  operator domain: ('Z -> 'Y) -> 'Z operand: ('Z -> 'Y) -> 'Z -> 'Y in
   expression: (S I) I
- S K I;
stdIn:27.1-27.6 Warning: type vars not generalized because of
   value restriction are instantiated to dummy types (X1,X2,...)
val it = fn : ?.X1 -> ?.X1
- S K I 1;
val it = 1 : int
- S K I 21;
val it = 21 : int
- val T = S K;
stdIn:31.1-31.12 Warning: type vars not generalized because of
   value restriction are instantiated to dummy types (X1,X2,...)
val T = fn : (?.X1 -> ?.X2) -> ?.X1 -> ?.X1
- val U = S K add1;
val U = fn : int -> int
- U 21;
val it = 21 : int
- U 234;
val it = 234 : int
- ^D
```